23 Algebraic Manipulation

Student Text

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23.1 Simultaneous Linear Equations

A pair of equations which use both terms at the same time, such as

$$x + 2y = 8$$
$$2x + y = 7$$

are known as a pair of *simultaneous* equations. It is a straightforward process to manipulate these equations into *one* linear equation by eliminating one of the unknowns (x or y). The equation can then be readily solved.

Worked Example 1

Solve the pair of simultaneous equations

$$x + 2y = 8$$
$$2x + y = 7$$

Solution

First it is helpful to label the equations (1) and (2).

$$x + 2y = 8 \qquad (1)$$
$$2x + y = 7 \qquad (2)$$

Equation (1) is multiplied by 2, so that it contains the same number of x's as equation (2).

Let the new equation be labelled (3).

$$2x + 4y = 16 \qquad (3) \qquad [2 \times (1)]$$
$$2x + y = 7 \qquad (2)$$

Equation (2) is now subtracted from equation (3).

$$2x + 4y = 16 \qquad (3)$$
$$2x + y = 7 \qquad (2)$$
$$3y = 9 \qquad (3) - (2)$$

Solving $3y = 9$ gives $y = 3$.

This value of y can now be substituted into equation (1) to give:

$$x + 2 \times 3 = 8$$
$$x + 6 = 8$$

Solving this gives $x = 2$. So the solution to the equation is $x = 2$, $y = 3$.

Worked Example 2

Solve the simultaneous equations

$$3x + 5y = 2$$
$$-4x + 7y = -30$$

Solution

First label the equations (1) and (2) as shown below.

$$3x + 5y = 2 \qquad (1)$$
$$-4x + 7y = -30 \qquad (2)$$

Then multiply equation (1) by 4 and equation (2) by 3 to make the number of x's in both equations the same.

$$12x + 20y = 8 \qquad (3) \qquad [4 \times (1)]$$
$$-12x + 21y = -90 \qquad (4) \qquad [3 \times (2)]$$

Now add together equations (3) and (4) to give

$$12x + 20y = 8 \qquad (3)$$
$$-12x + 21y = -90 \qquad (4)$$
$$41y = -82 \qquad (3) + (4)$$

Solving the equation $41y = -82$ gives $y = -2$.

This value for y can be substituted into equation (1) to give

$$3x + 5 \times (-2) = 2$$
or
$$3x - 10 = 2$$

Solving this equation gives:

$$3x - 10 = 2$$
$$3x = 12$$
$$x = \frac{12}{3}$$
$$= 4$$

So the solution is $x = 4$ and $y = -2$.

Note

It is a good idea to check that solutions are correct by substituting these values back into the original equations. Here,

$$3 \times 4 + 5 \times (-2) = 2$$
and
$$-4 \times 4 + 7 \times (-2) = -30$$

You must check *both* equations to make sure that you have the correct answer.

Worked Example 3

Denise sells 300 tickets for a concert. Some tickets are sold to adults at \$5 each and some are sold to children at \$4 each. If she collects in \$1444 in ticket sales, how many tickets have been sold to adults and how many to children?

Solution

Let x = number of adults' tickets

and y = number of children's tickets.

She has sold 300 tickets, so

$$x + y = 300$$

The value of the adult tickets sold is \$5x, and the value of the children's tickets is \$4y.

As the value of all the tickets sold is \$1444, then

$$5x + 4y = 1444$$

The two simultaneous equations

$$x + y = 300 \qquad (1)$$

$$5x + 4y = 1444 \qquad (2)$$

can now be solved. First multiply equation (1) by 5 and subtract equation (2) to give

$$5x + 5y = 1500 \qquad (3) \qquad [5 \times (1)]$$

$$5x + 4y = 1444 \qquad (2)$$

$$y = 56 \qquad (3) - (2)$$

This value can then be substituted into equation (1) to give

$$x + 56 = 300$$

or

$$x = 244$$

So the solution is $x = 244$ and $y = 56$. That is, 244 adults' tickets and 56 children's tickets have been sold.

Investigation

Consider the following simultaneous equations.

$$2x + y = 6 \qquad (1)$$

$$x = 1 - \tfrac{1}{2}y \qquad (2)$$

If (2) is substituted for x into (1), then

$$2\left(1 - \tfrac{1}{2}y\right) + y = 6$$

$$2 - y + y = 6$$

$$2 = 6$$

Find out where the problem lies.

Exercises

1. Solve each pair of simultaneous equations.

(a) $x + 2y = 5$, $3x + y = 5$

(b) $3x + 2y = 19$, $x + 5y = 15$

(c) $x - 2y = 4$, $4x + 3y = 49$

(d) $2x + 3y = 14$, $5x + 2y = 24$

(e) $3x + 4y = 2$, $7x - 5y = 9$

(f) $4x + 2y = 16$, $-3x + 2y = -19$

(g) $5x + y = 2$, $-4x + 3y = 44$

(h) $6x - 4y = 12$, $-9x + 2y = -66$

(i) $7x - 2y = 23$, $3x + 4y = 39$

(j) $8x + 4y = 7$, $-12x + 8y = -6$

(k) $4x - 2y = -0.1$, $5x + 2y = 1.5$

(l) $6x - 5y = 41$, $4x + 15y = 31$

(m) $-2x + 5y = 14$, $10x + 7y = 26$

(n) $8x + 5y = -29$, $3x - 7y = -2$

(o) $6x - 5y = -14$, $18x - 4y = 6$

(p) $6x - 8y = -2$

(q) $\frac{1}{2}x - \frac{1}{4}y = 0$, $\frac{1}{3}x + \frac{2}{3}y = 10$

(r) $\frac{1}{5}x - \frac{1}{10}y = -1$, $\frac{1}{4}x + \frac{1}{2}y = 10$

2. Find the coordinates of the point of intersection of the lines:

(a) $x + y = 8$ and $y = 2x - 1$

(b) $x + y = 10$ and $y = 2x + 1$

(c) $x + y = 4$ and $y = 2 - \frac{x}{10}$

3. Describe the problems you encounter when you try to solve the simultaneous equations:

$$3x - 2y = 8$$
$$9x - 6y = 2$$

4. (a) Check that $x = 2$ and $y = 5$ is a solution of both the equations below.

$$x + 2y = 12$$
$$3x + 6y = 36$$

(b) Try to solve the equations. What happens?

(c) Write both equations in the form $y = \ldots$ and comment on the equations you obtain.

5. Dexter rows a boat in a river where there is a steady current. He travels at 1.3 mph upstream and 3.7 mph downstream.

   Use a pair of simultaneous equations to find v, the speed of the boat in still water, and c, the speed of the current.

6. A machine sells tickets for travel on a tram system. A single ticket costs \$1 and a return ticket costs \$2. In one day, the machine sells 100 tickets and takes \$172. How many of each type of ticket were sold?

7. A shopkeeper takes 200 notes to a bank. They are a mixture of J\$50 and J\$100 notes. The shopkeeper claims that there is J\$13 950.

   (a) Find out how many of each type of note there should be.

   (b) In fact the value of the notes turns out to be J\$13 900. How many of each type of note does this mean there should be? What error do you think the shopkeeper made?

8. A group of people boarded a bus at a bus station. The fare to the first bus stop was \$3 and the fare to the second bus stop was \$5. When the bus left the first stop, there were 38 people on the bus and the driver had collected \$238 in fares. How many people got off the bus at the first stop?

9. When Aleah travels from the UK to the USA for a holiday she leaves at 1.00 pm local time and lands at 5.00 pm local time on the same day. On the return journey she leaves at 8.00 pm local time and lands at 10.00 am local time the next day.

   Find the length of the flight in hours and the time difference between the UK and the part of the USA that Aleah visited.

10. Solve the simultaneous equations,

$$2x + 3y = 23$$
$$x + y = 4$$

11. Solve the simultaneous equations,

$$2a + 4c = 13$$
$$a + 3c = 8$$

12. The height, h metres, of a sky rocket t seconds after being launched is given by the formula

$$h = at^2 + bt + 2$$

   where a and b are constants. The heights of the rocket above the ground at two different times are given in the table below.

t (seconds)	1	2
h (metres)	37	62

(a) At what height above the ground is the rocket launched?

(b) (i) Use the table of values to show that

$$a + b = 35$$

and $$4a + 2b = 60.$$

(ii) Solve these simultaneous equations to find the value of a and the value of b.

(c) What was the height of the sky rocket $7\frac{1}{2}$ seconds after it was launched?

23.2 Expanding Brackets

Equations or formulae may contain brackets, for example

$$P = 8(x - 7) \quad \text{or} \quad y = (2x - 3)(x - 8)$$

Removing brackets from these types of expressions is known as *expanding*.

When evaluating or manipulating expressions it is sometimes important to be able to use the original equation, whereas at other times the expanded version might be helpful.

The reverse of expanding, the process of factorisation, is even more important and becomes easier once you have gained confidence in expanding brackets.

Worked Example 1

Expand $2x(5x - 8)$.

Solution

$$2x(5x - 8) = 2x \times 5x - 2x \times 8$$

$$= 10x^2 - 16x$$

Worked Example 2

Expand $-4(x - 6)$.

Solution

$$-4(x - 6) = -4 \times x + (-4) \times (-6)$$

$$= -4x + 24$$

Sometimes situations will arise where a bracket has to be multiplied by another bracket, as in the next example.

Worked Example 3

Expand $(x + 2)(x + 5)$.

Solution

The first bracket $(x + 2)$ is split so that each if its terms, (x) and $(+ 2)$, can multiply the other bracket.

$$\begin{aligned}(x+2)(x+5) &= x(x+5) + 2(x+5)\\ &= x^2 + 5x + 2x + 10\\ &= x^2 + 7x + 10 \qquad \text{(collecting like terms)}\end{aligned}$$

Worked Example 4

Expand $(4x - 3)(2x - 7)$

Solution

Splitting the first bracket and multiplying the other bracket by each of its terms gives,

$$\begin{aligned}(4x-3)(2x-7) &= 4x(2x-7) - 3(2x-7)\\ &= 8x^2 - 28x - 6x + 21\\ &= 8x^2 - 34x + 21\end{aligned}$$

Worked Example 5

Expand $(x + 6)^2$.

Solution

First note that

$$(x+6)^2 = (x+6)(x+6)$$

Then the brackets can be expanded.

$$\begin{aligned}(x+6)(x+6) &= x(x+6) + 6(x+6)\\ &= x^2 + 6x + 6x + 36\\ &= x^2 + 12x + 36\end{aligned}$$

Worked Example 6

Expand $(3x - 2n)(4x + 5n)$.

Solution

$$\begin{aligned}(3x-2n)(4x+5n) &= 3x(4x+5n) - 2n(4x+5n)\\ &= 12x^2 + 15xn - 8nx - 10n^2\\ &= 12x^2 + 7xn - 10n^2\end{aligned}$$

Note that *xn* is the same as *nx*.

Exercises

1. Simplify each expression below, expanding the brackets as necessary.

(a) $x(x+1)$ (b) $-x(3x-2)$ (c) $3a(2a-5)$

(d) $x^2(1-x)$ (e) $4x(x-7)$ (f) $5+3(x+2)$

(g) $4x+x(x-6)$ (h) $x(1+x^2)$ (i) $5-4(x^2+2)$

(j) $x^2-x(x+5)$ (k) $x^2-4x(5-x)$ (l) $x^2+4(x^2-2)$

(m) $x^3+x^2(5-2x)$ (n) $4(x+4)-x$ (o) $5x-x(3x-2)$

2. Expand and simplify each of the following.

(a) $(x+1)(x+6)$ (b) $(x+4)(x+7)$ (c) $(x-2)(x+8)$

(d) $(x-1)(x+4)$ (e) $(x-7)(x-1)$ (f) $(2x+1)(3x-1)$

(g) $(4x+3)(2x+7)$ (h) $(5x-2)(2x+3)$ (i) $(6x-1)(4x+3)$

(j) $(3x-1)(4x-3)$ (k) $(5x-6)(3x-8)$ (l) $(3a-7)(5a+2)$

(m) $(4a-5)(3a-5)$ (n) $(6n+1)(4n-2)$ (o) $(4x+3)(5x-8)$

3. Expand and simplify each of the following.

(a) $(x+1)^2$ (b) $(x-1)^2$ (c) $(x+8)^2$

(d) $(2x+1)^2$ (e) $(3x+4)^2$ (f) $(6x-1)^2$

(g) $(2x-5)^2$ (h) $(6x-7)^2$ (i) $(5x+9)^2$

4. Expand and simplify, whenever possible, the following.

(a) $(a+b)(c+d)$ (b) $(a+c)(2a+c)$ (c) $(3a+d)(a-5d)$

(d) $(4x-y)(3x+y)$ (e) $(4a+d)(a+3d)$ (f) $(2a+b)(a+4c)$

(g) $(6x-y)(y-3x)$ (h) $(p+q)(q-2p)$ (i) $(5x+y)(9x-2y)$

(j) $(4x-2)(x+y)$ (k) $(2a+b)(2a-c)$ (l) $(4x-5y)(x-6y)$

(m) $(p-2q)(p+2q)$ (n) $(5a+2b)(a-3b)$ (o) $(5x-6y)(2x-3y)$

5. (a) Expand and simplify

(i) $(x+1)(x-1)$ (ii) $(x+4)(x-4)$ (iii) $(x+6)(x-6)$

(b) Without expanding the brackets, write down what you would expect to get if you expanded

$$(x+5)(x-5)$$

(c) Expand and simplify

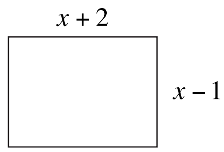
$$(a + b)(a - b)$$

(d) What would you expect to obtain if you expanded

$$(5x + 2)(5x - 2)?$$

6. Find the area of each rectangle.

(a)



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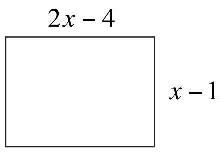

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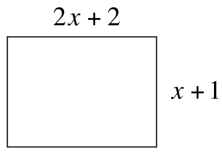

7. Expand and simplify each expression.

(a) $x(x + 2)(x - 5)$ (b) $(x + 2)(x^2 + 2x - 8)$

(c) $(x + 1)(x - 2)(x + 3)$ (d) $(2x + 1)(3x - 4)(x + 7)$

(e) $(x + 1)^3$ (f) $(x - 4)^3$

23.3 Linear Factorisation

The process of *removing* brackets is known as *expanding*. The reverse process is known as *factorisation*, where an expression is rewritten as a product of terms.

To factorise an expression it is necessary to identify numbers or variables which are factors common to all the terms.

Worked Example 1

Factorise $6x + 8$.

Solution

Both terms ($6x$) and (8) can be divided by 2, so the expression is factorised as

$$\begin{aligned} 6x + 8 &= (2 \times 3x) + (2 \times 4) \\ &= 2(3x + 4) \end{aligned}$$

Worked Example 2

Factorise $12a - 16$.

Solution

Here the largest number by which both terms, (12a) and (16), can be divided is 4.

$$\begin{aligned} 12a - 16 &= (4 \times 3a) - (4 \times 4) \\ &= 4(3a - 4) \end{aligned}$$

Worked Example 3

Factorise $4x^2 - 8x$.

Solution

Here 4 is the largest number that will divide both terms, but each term can also be divided by x, so $4x$ is the factor common to both terms.

$$4x^2 - 8x = (4x \times x) - (4x \times 2)$$
$$= 4x(x - 2)$$

Exercises

1. Copy and complete each of the following.

(a)	$5x + 10 = ?(x + 2)$	(b)	$6x - 8 = ?(3x - 4)$
(c)	$15x + 25 = ?(3x + 5)$	(d)	$12x + 8 = 4(? + ?)$
(e)	$18 - 6n = 6(? - ?)$	(f)	$6x - 21 = 3(? - ?)$
(g)	$16a + 24 = 8(? + ?)$	(h)	$33x - 9 = 3(? - ?)$

2. Factorise each of the following expressions.

(a)	$6x + 24$	(b)	$5x - 20$	(c)	$16 - 8x$
(d)	$8n + 12$	(e)	$12x - 14$	(f)	$3a - 24$
(g)	$11x - 66$	(h)	$10 + 25x$	(i)	$100x - 40$
(j)	$50 - 40x$	(k)	$6x - 30$	(l)	$5y - 45$
(m)	$12 + 36x$	(n)	$16x + 32$	(o)	$27x - 33$

3. Complete a copy of each of the following.

(a)	$x^2 + x = ?(x + 1)$	(b)	$x^2 + 2x = ?(x + 2)$
(c)	$2a^2 - 5a = ?(2a - 5)$	(d)	$4x^2 + x = x(? + ?)$
(e)	$x^2 + 4x = x(? + ?)$	(f)	$xa + xb = x(? + ?)$
(g)	$6x^2 + 3x = 3x(? + ?)$	(h)	$4x^2 - 2ax = 2x(? - ?)$

4. Factorise each of the following expressions.

(a)	$5x^2 + x$	(b)	$a^2 + 3a$	(c)	$5n^2 + 2n$
(d)	$6n^2 + 3n$	(e)	$5n^2 - 10n$	(f)	$3x^2 + 6x$
(g)	$15x^2 + 30x$	(h)	$14x^2 + 21x$	(i)	$16x^2 + 24x$
(j)	$30x^2 - 18x$	(k)	$5 + 5n^2$	(l)	$10n^2 - 15$
(m)	$3n^3 + 9n$	(n)	$9x^2 - 27x$	(o)	$10x^3 - 5x^2$

5. Factorise each of the following expressions.

(a) $ax + ax^2$ (b) $bx + cx^2$ (c) $2pq - 4rq$

(d) $15xy - 5y^2$ (e) $16pq + 24p^2$ (f) $6x^2 + 18xy$

(g) $3p^2 - 9px$ (h) $24px + 56x^2$ (i) $16x^2y - 18xy^2$

6. For each factorisation shown below, state whether it can be factorised further. If the answer is yes, give the complete factorisation.

(a) $6x^2 + 4x = 2\left(3x^2 + 2x\right)$ (b) $16x^3 + 8x^2 = 8x\left(2x^2 + x\right)$

(c) $5x^2 - 60x = 5x\left(x - 12\right)$ (d) $3x^2y - 18xy^2 = 3x\left(xy - 6y^2\right)$

23.4 Quadratic Factorisation

It is also possible to factorise expressions such as

$$x^2 + 5x + 6$$

to obtain

$$\left(x + 2\right)\left(x + 3\right)$$

First consider what happens when two brackets are multiplied together. For example,

$$\left(x + 2\right)\left(x + 3\right) = x^2 + 5x + 6 .$$

Note that the 5 is given by $2 + 3$ and the 6 is given by 2×3.

When factorising a quadratic like this we need to find two numbers which, when added together, give one number and when multiplied together give the other number.

For example, when factorising

$$x^2 + 8x + 12$$

we need two numbers which give 12 when multiplied and 8 when added. These are, of course, 2 and 6. Hence $x^2 + 8x + 12 = \left(x + 2\right)\left(x + 6\right)$.

Worked Example 1

Factorise $x^2 + 9x + 20$.

Solution

The solution will be of the form

$$\left(x + a\right)\left(x + b\right)$$

where $a \times b = 20$ and $a + b = 9$.

You may immediately see that the two numbers are 4 and 5. However, it will not always be obvious. A helpful approach is to write the possible pairs of numbers which multiply to give 20.

$$x^2 + 9x + 20 = (x + \quad)(x + \quad)$$

1	20
2	10
4	5

It is then easy to see that only the third pair of numbers add up to 9. So

$$x^2 + 9x + 20 = (x + 4)(x + 5)$$

Worked Example 2

Factorise $x^2 - 3x - 10$.

Solution

The solution will be of the form $(x \quad)(x \quad)$. Considering the ways of obtaining -10 by multiplication gives

$$x^2 - 3x - 10 = (x \quad)(x \quad)$$

-1	$+10$
-2	$+5$
$+1$	-10
$+2$	-5

Only the fourth possibility gives a total of -3 when the two terms are added, so

$$x^2 - 3x - 10 = (x + 2)(x - 5)$$

Worked Example 3

Factorise $x^2 - 5x + 6$.

Solution

The solution will be of the form $(x \quad)(x \quad)$. Considering ways of obtaining $+6$ (including negative factors since the x component has a negative coefficient) gives:

$$x^2 - 5x + 6 = (x \quad)(x \quad)$$

$+6$	$+1$
$+3$	$+2$
-6	-1
-3	-2

The last of these gives a total of -5 when the two terms are added, so

$$x^2 - 5x + 6 = (x - 3)(x - 2)$$

Worked Example 4

Factorise $2x^2 - x - 3$.

Solution

The solution will be of the form $(2x \quad)(x \quad)$ to give the $2x^2$ term. Considering the ways of obtaining -3 gives:

$$2x^2 - x - 3 = (2x \quad)(x \quad)$$

-1	$+3$
$+1$	-3
$+3$	-1
-3	$+1$

Note that these will be multiplied by the 2 in the $2x$ term

From the last of these we can obtain the middle term,

$$(-3 \times x) + (1 \times 2x) = -3x + 2x$$
$$= -x$$

so

$$2x^2 - x - 3 = (2x - 3)(x + 1)$$

You can check that these brackets multiply out to give the original expression.

Worked Example 5

Factorise completely

(a) $x^2 - xy$

(b) $e^2 - 1$

(c) $5p^2 + 9pq - 2q^2$ *(CXC)*

Solution

(a) $x^2 - xy = x(x - y)$

(b) $e^2 - 1 = (e - 1)(e + 1)$

(c) $5p^2 + 9pq - 2q^2 = (5p - q)(p + 2q)$

(The other possibilities which do not give the correct RHS are

$$(5p + q)(p - 2q) = 5p^2 - 9pq - 2q^2$$

$$(5p + 2q)(p - q) = 5p^2 - 3pq - 2q^2$$

$$(5p - 2q)(p + q) = 5p^2 + 3pq - 2q^2 \text{)}$$

Note

It is often a good idea to check the answers you obtain by expanding the brackets.

You may remember that

$$(a - b)(a + b) = a^2 - b^2$$

This result is known as the *difference between two squares* and can be used to factorise some expressions.

Worked Example 6

Factorise the following using the difference between two squares result.

(a) $x^2 - 9$ (b) $4x^2 - 25$

Solution

(a) $x^2 - 9 = x^2 - 3^2$

$= (x + 3)(x - 3)$

(b) $4x^2 - 25 = (2x)^2 - 5^2$

$= (2x - 5)(2x + 5)$

Exercises

1. Factorise each of the following expressions.

(a)	$x^2 + 4x + 4$	(b)	$x^2 + 7x + 12$	(c)	$x^2 + 6x + 8$
(d)	$x^2 + 7x + 6$	(e)	$x^2 + 10x + 16$	(f)	$x^2 + 4x + 3$
(g)	$x^2 + 8x + 15$	(h)	$x^2 + 3x + 2$	(i)	$x^2 + 5x + 4$
(j)	$x^2 + 11x + 24$	(k)	$x^2 + 12x + 11$	(l)	$x^2 + 15x + 56$
(m)	$x^2 + 6x + 9$	(n)	$x^2 + 7x + 10$	(o)	$x^2 + 9x + 14$
(p)	$x^2 + 11x + 30$	(q)	$x^2 + 9x + 8$	(r)	$x^2 + 12x + 32$

2. Factorise the following expressions.

(a)	$x^2 + x - 2$	(b)	$x^2 - x - 12$	(c)	$x^2 - 3x - 10$
(d)	$x^2 + 4x - 5$	(e)	$x^2 - 5x - 14$	(f)	$x^2 - 2x - 8$
(g)	$x^2 + 2x - 15$	(h)	$x^2 - 3x + 2$	(i)	$x^2 - 9x + 20$
(j)	$x^2 - 10x + 21$	(k)	$x^2 - 9x + 14$	(l)	$x^2 - 7x + 10$
(m)	$x^2 - 6x - 16$	(n)	$x^2 - 17x + 72$	(o)	$x^2 - 5x - 24$

3. Factorise each of the following using the difference of two squares result.

(a) $x^2 - 1$ (b) $x^2 - 16$ (c) $x^2 - 81$

(d) $9x^2 - 4$ (e) $16x^2 - 36$ (f) $4x^2 - 100$

(g) $x^4 - 100$ (h) $x^4 - 4$ (i) $4x^4 - 9$

4. Complete each of the following factorisations.

(a) $2x^2 + 3x + 1 = (2x + 1)(x + ?)$

(b) $3x^2 + 7x + 2 = (3x + 1)(x + ?)$

(c) $2x^2 + 5x + 3 = (2x + ?)(x + 1)$

(d) $3x^2 + 14x + 8 = (3x + ?)(x + 4)$

(e) $2x^2 + 9x - 5 = (2x - 1)(x + ?)$

(f) $4x^2 - 5x - 6 = (4x + ?)(x - ?)$

(g) $3x^2 + x - 10 = (3x - ?)(x + ?)$

(h) $3x^2 - 23x + 14 = (3x - ?)(x - ?)$

(i) $6x^2 + 17x + 5 = (3x + ?)(2x + ?)$

(j) $8x^2 - 6x + 1 = (4x - ?)(2x - ?)$

5. Factorise each of the following expressions.

(a) $3x^2 - 2x - 1$ (b) $3x^2 + 4x + 1$ (c) $2x^2 + 5x + 2$

(d) $3x^2 + 8x + 4$ (e) $3x^2 + 8x - 3$ (f) $4x^2 - 11x - 3$

(g) $5x^2 + 3x - 2$ (h) $3x^2 - 8x + 4$ (i) $5x^2 + 13x - 6$

(j) $6x^2 + 5x + 1$ (k) $6x^2 - 7x + 2$ (l) $10x^2 - 3x - 1$

(m) $8x^2 + 10x - 3$ (n) $6x^2 + 19x - 7$ (o) $6x^2 - 17x + 12$

6. (a) Expand $(2x + 1)(x + 4)$.

(b) Factorise completely $4x^2 - 6x$.

7. (a) Factorise completely $12p^2q - 15pq^2$.

(b) Expand and simplify $(2x - 3)(x + 5)$.

(c) The cost, C pence, of printing n party invitations is given by

$$C = 120 + 40n$$

Find a formula for n in terms of C.

8. Factorise

(a) $5a^2b + ab^2$

(b) $9k^2 - 1$

(c) $2y^2 - 5y + 2$ *(CXC)*

9. Factorise

(a) $4g^2 - f^2$

(b) $tm - 3t + 2pm - 6p$ *(CXC)*

10. Factorise completely

(a) $7mp^2 + 14m^2p$

(b) $2y^2 - 11y + 15$ *(CXC)*

Challenge!

Which area is larger and by how much – a half km square or a half square km?