STRAND I: Geometry and Trigonometry

Unit 32 *Angles, Circles and Tangents*

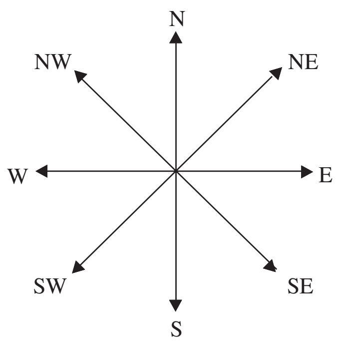
Student Text

Contents

32 Angles, Circles and Tangents

32.1 Compass Bearings

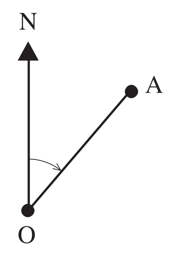
When describing a direction, the points of a compass can be useful, e.g. S (south) or SW (south west).



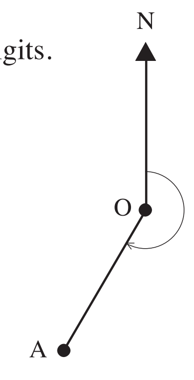
A *bearing* can also be used, often in navigation and by people walking on rough or open moorland or hills.

Note

Bearings are always measured clockwise from north and use 3 digits.



The bearing of A from O is $050°$.



The bearing of A from O is $210°$.

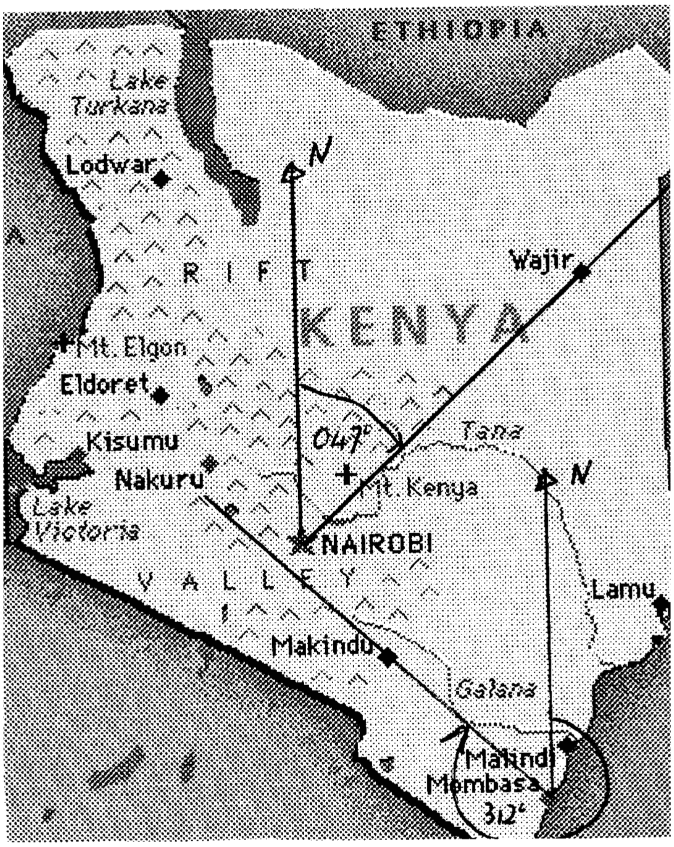
Worked Example 1

On a map of Kenya, find the bearings of

(a) Wajir from Nairobi

(b) Makindu from Mombasa.

Solution



Map of Kenya

(a) First draw in a north line at Nairobi and another line from Nairobi to Wajir.

Then measure the angle clockwise from north to the second line. In this case the angle is $47°$ so the bearing is $047°$.

(b) Draw a north line at Mombassa and a line from Mombassa to Makindu.
The bearing can then be measured as $312°$.

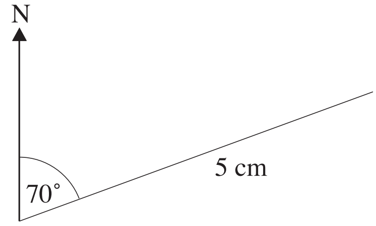
Worked Example 2

A boat sails for 500 miles on a bearing of $070°$ and then sails a further 700 nautical miles on a bearing of $200°$. Find the distance of the boat from its starting point and the bearing that would have taken it straight there.

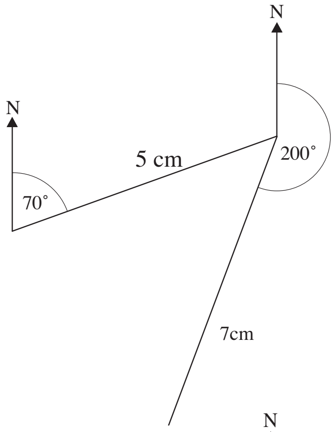
Solution

To find the solution use a scale drawing.

1. Draw a north arrow at the starting point.
2. Measure an angle of $70°$ from North.
3. Draw a line 5 cm long.
   (1 cm represents 100 nautical miles.)



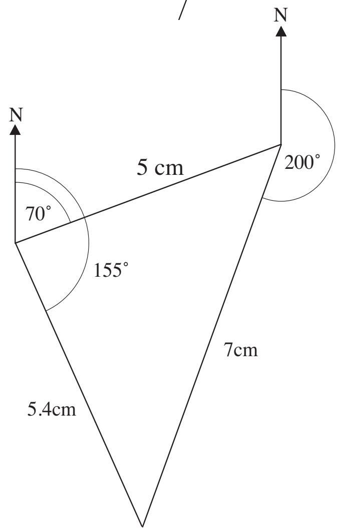
4. Draw a second north arrow and measure an angle of $200°$.



5. Draw a line 7 cm long.

6. Join the final point to the starting point and measure the distance.

   It is 5.4 cm, which represents 540 nautical miles.



7. The bearing can also be measured as $155°$.

Worked Example 3

A ship leaves port *P* and sails to port *Q* on a bearing of $124°$. From *Q*, the ship travels to port *R* on a bearing of $320°$.

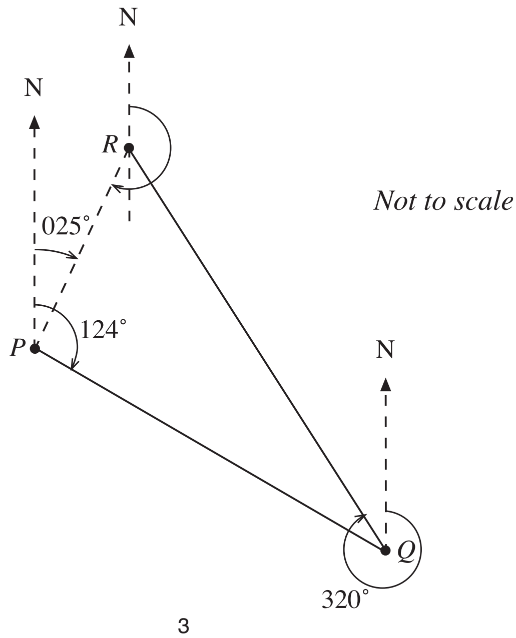
Given that the bearing of *R* from *P* is $025°$:

(a) Draw a carefully labelled diagram to represent the journey of the ship.

(b) Determine the bearing of *P* from *R*.

Solution

(a)



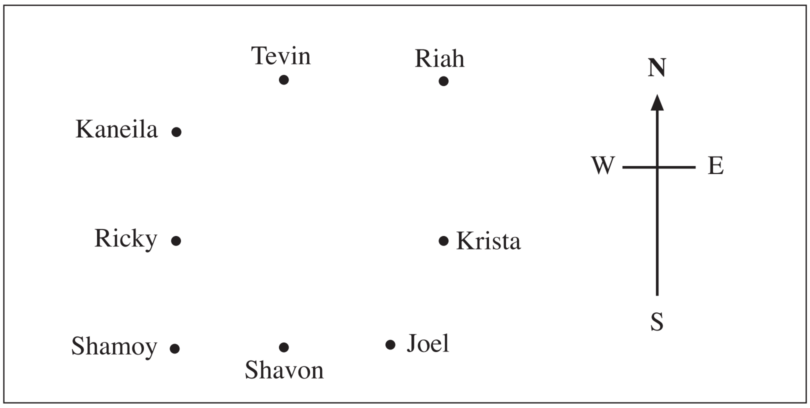
(b) Bearing of *P* from *R* is

$180° + 025° = 205°$

Bearings are used again in section 4 of Unit 34, when you calculate distances between points.

Exercises

1. The diagram shows the positions of 8 friends.



(a) Who is directly south of Riah?

(b) If Kaneila walks SE, whom will she meet?

(c) If Riah walks SW, whom will she meet?

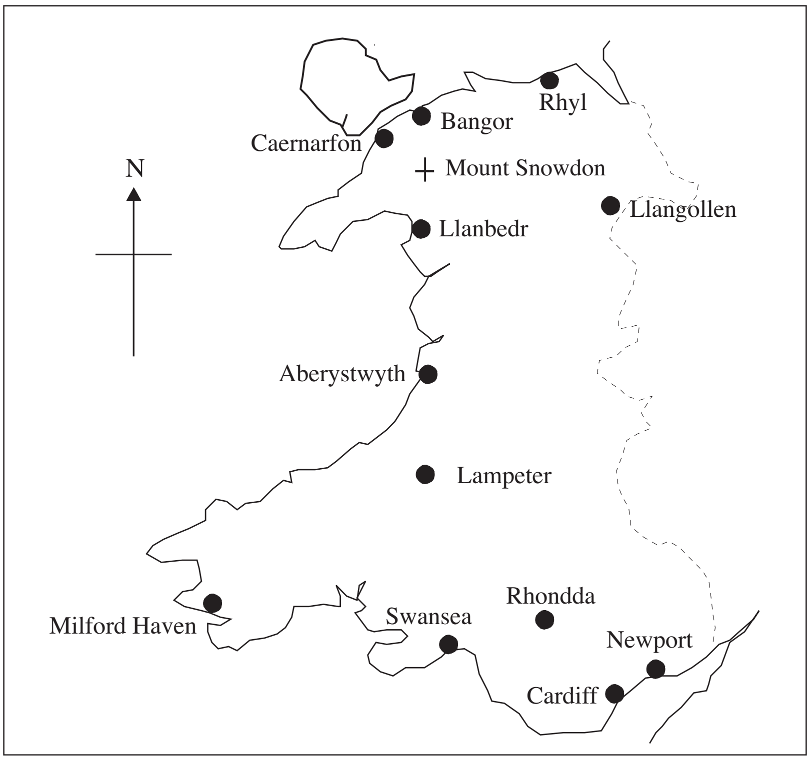
(d) Who is directly west of Krista?

(e) Who is NW of Krista?

(f) Who will Shavon meet if he walks NW?

(g) In what direction should Shamoy walk to find Riah?

2. The map shows some towns and cities in the UK.

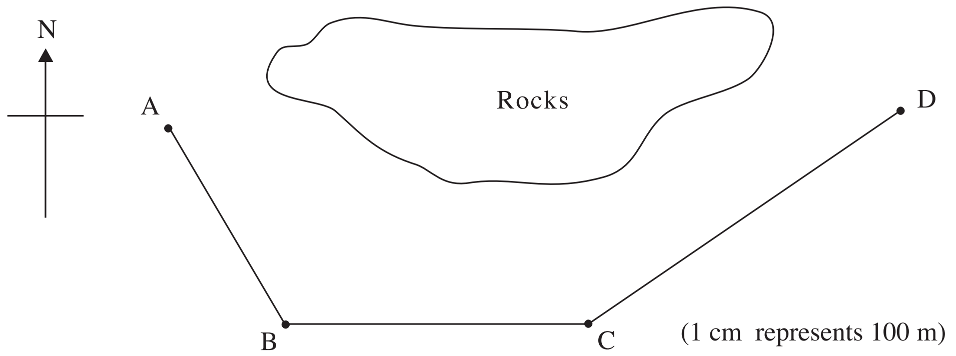


Write down the bearing of each of the following places from Mount Snowdon.

(a) Llangollen (b) Newport (c) Swansea

(d) Bangor (e) Milford Haven (f) Aberystwyth

3. In order to avoid an area of dangerous rocks a yacht sails as shown in the diagram.

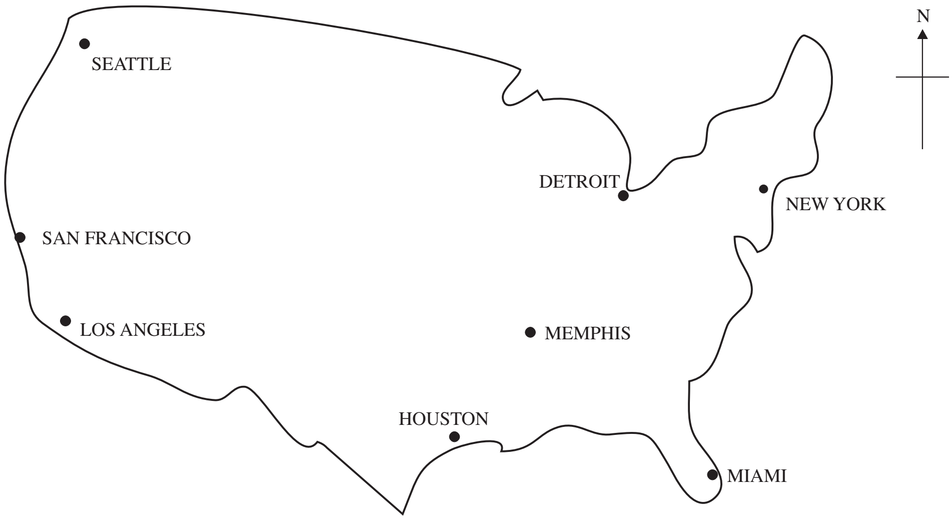


(a) Find the bearing of the yacht as it sails from

(i) A to B (ii) B to C (iii) C to D.

(b) How much further does the yacht travel to avoid the rocks?

4. A roughly sketched map of the USA is shown below.



Find the bearings of:

(a) Miami from Memphis. (b) Detroit from New York

(c) Los Angeles from Detroit. (d) Seattle from Houston.

(e) Memphis from San Francisco.

5. Use a scale drawing to find the answers to each of the following problems.

   (a) If a man walks 700 m on a bearing of $040°$, how far north and how far east is he from his starting point?

   (b) A dog runs 50 m on a bearing of $230°$ and then runs north until he is west of his starting point. How far is the dog from its starting point?

   (c) A helicopter flies 80 km north and then 20 km SW. What bearing would have taken the helicopter directly to its final position?
   How far is the helicopter from its starting point?

   (d) A boat travels 500 m NE and then 500 m south. What bearing would take the boat back to its starting point?

   (e) A plane flies 300 km west and then a further 200 km SW. It then returns directly to its starting point.
   On what bearing should it fly and how far does it have to travel?

6. Use a scale drawing to illustrate each of the following journeys.
   Describe the return journey in each case using a single bearing and distance.

   (a) 120 m on $090°$ followed by 120 m on $180°$.

   (b) 500 km on $045°$ followed by 200 km on $270°$.

   (c) 300 km on $220°$ followed by 300 km on $170°$.

   (d) 25 km on $330°$ followed by 30 km on 170.

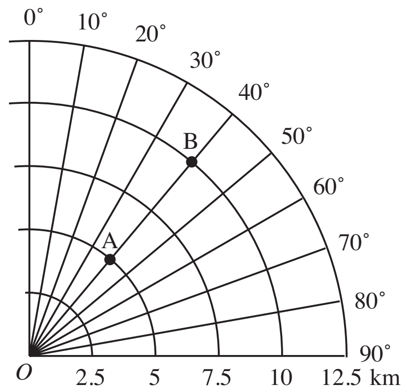
   (e) 10 km on $160°$ followed by 2 km on $300°$.

   (f) 30 km on $120°$ followed by 30 km on $270°$.

   (g) 1000 m on $050°$ followed by 1200 m on $310°$.

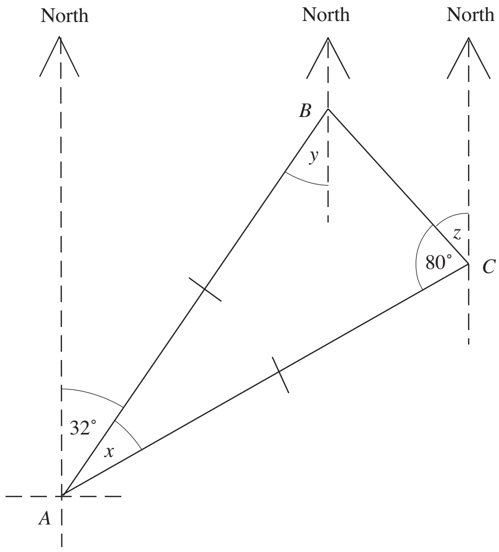
7. A ship sails from a point A to another point B, 8000 m due east of A.
   It then sails in another direction and arrives at a point C, 10 000 m SE of A.
   On what bearing did the ship sail on the second stage of the journey and how far did it travel?

8. The position of ship A from *O* is 5 km on a bearing of $040°$. The position of ship B from *O* is 10 km on a bearing fo $040°$.



What is the position of ship A from ship B?

9.



The diagram is not drawn to scale.

The diagram shows the position of three places, A, B and C.
AB is the same length as AC.

(a) (i) Calculate the size of the angle marked *x*.

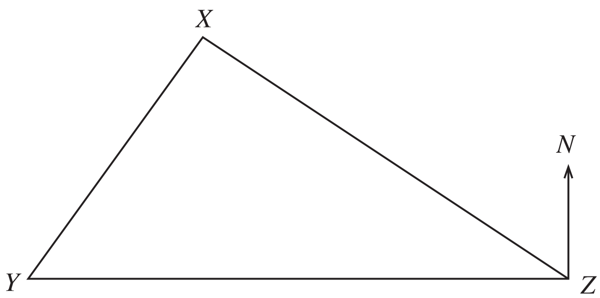
(ii) Explain why the angle marked *y* is equal to $32°$.

(iii) Calculate the size of the angle marked *z*.

(b) Use your answers to (a) to calculate the bearing of

(i) C from A (ii) A from B (iii) B from C.

10. The figure below, **not drawn to scale**, represents the journey of an aircraft flying from *Y* to *X* and then from *X* to *Z*.



The bearing of *X* from *Y* is $035°$.

The bearing of *Z* from *X* is $125°$.

Z is due east of *Y*.

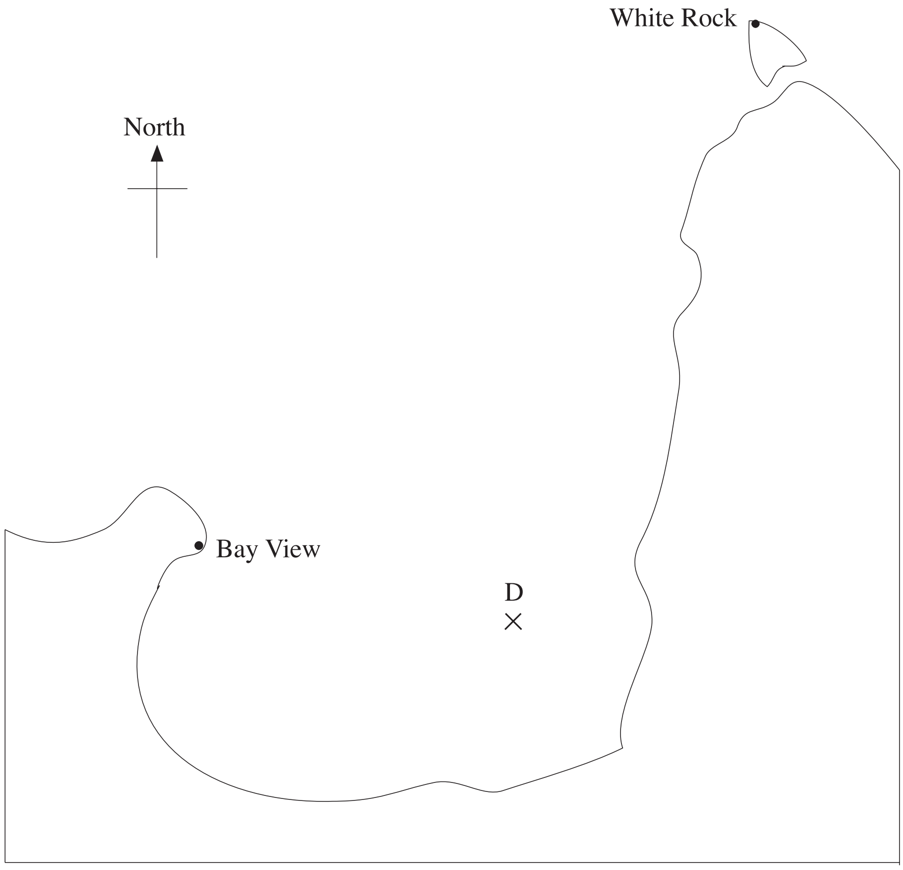
(a) Copy and complete the diagram, showing CLEARLY the bearings $035°$ and $125°$.

(b) Determine the size of angle *YXZ*.

(c) Determine the size of the angle *XZY*.

(CXC)

11. The diagram shows a bay in which yachts are moored.
The diagram has been drawn to a scale of 5 cm to 1 km.



(a) The yacht *Daresa* is moored at D.
Measure the bearing of this yacht from Bay View.

(b) The yacht *Wet-n-Windy* is moored 1.2 km from White Rock on a bearing of $210°$. Trace the diagram and mark with a cross the position of this yacht on the diagram.

Investigation

Draw a rectangle of any size. Use your ruler to locate the mid-points of the sides. Join these mid-points to form a new quadrilateral.

What is the name of the quadrilateral you have obtained?

Repeat the above by drawing

(a) a trapezium *(b) a parallelogram* *(c) a kite*

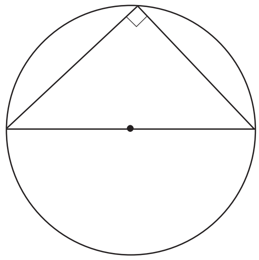
(d) a rhombus *(e) a quadrilateral of 4 unequal lengths.*

What conclusion can you draw from these?

32.2 Angles and Circles 1

The following results are true in any circle.

When a triangle is drawn in a semi-circle as shown, the angle on the perimeter is always a right angle.



Proof

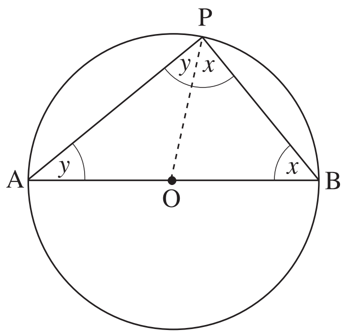
Join the centre, O, to the point, P, on the perimeter.

Since

$$OB = OP \qquad \text{(equal radii)}$$

then

$$\text{angle OBP} = \text{angle OPB} \qquad (= x, \text{ say})$$



Similarly, triangle AOP is also isosceles and

$$\text{angle OAP} = \text{angle APO} \qquad (= y, \text{ say}).$$

In triangle ABP, the sum of the angles must be $180°$.
Then

$$y + x + (x + y) = 180°$$

$$2x + 2y = 180° \qquad \text{(collecting like terms)}$$

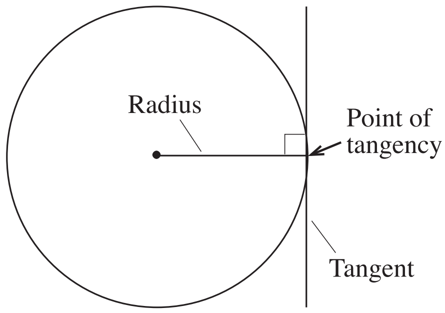
$$x + y = 90° \qquad (\div 2)$$

But angle APB $= x + y$, and this is a right angle.

A *tangent* is a line that touches only one point on the circumference of a circle.

This point is known as the *point of tangency*.

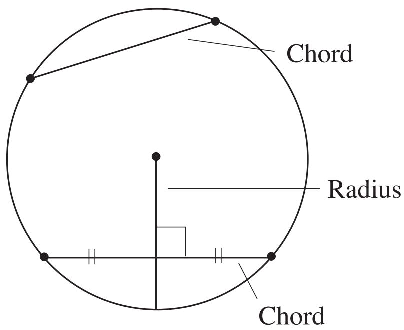
A tangent is always perpendicular to the radius of the circle.

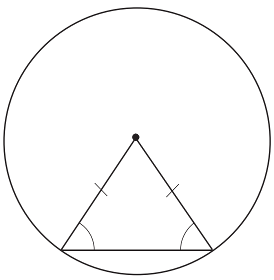


A *chord* is a line joining any two points on the circle.

The *perpendicular bisector* is a second line that divides the first line in half and is at right angles to it.

The perpendicular bisector of a chord is always a radius of the circle.

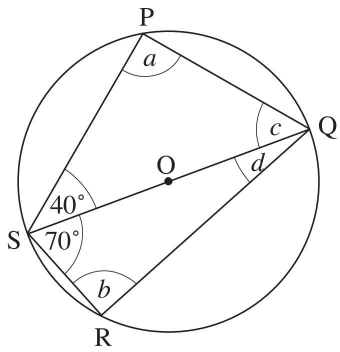




When the ends of a chord are joined to the centre of a circle, an isosceles triangle is formed, so the two angles marked are equal.

Worked Example 1

Find the angles marked with letters in the diagram, if O is the centre of the circle.



Solution

As both triangles are in semi-circles, angles a and b must each be $90°$.

The other angles can be found because the sum of the angles in each triangle is $180°$.

For the triangle PQS,

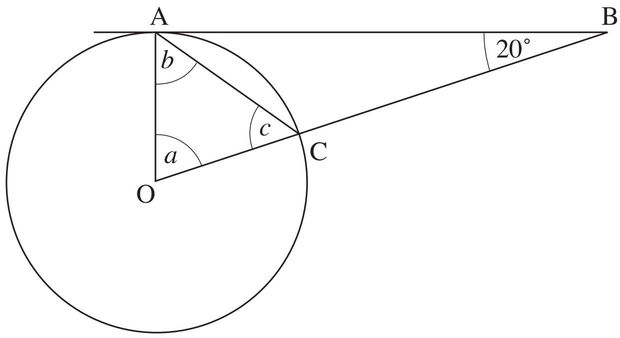
$$\begin{aligned} 40° + 90° + c &= 180° \\ c &= 180° - 130° \\ &= 50° \end{aligned}$$

For the triangle QRS,

$$\begin{aligned} 70° + 90° + d &= 180° \\ d &= 180° - 160° \\ &= 20° \end{aligned}$$

Worked Example 2

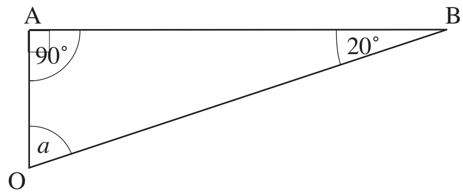
Find the angles a, b and c, if AB is a tangent and O is the centre of the circle.



Solution

First consider the triangle OAB. As OA is a radius and AB is a tangent, the angle between them is $90°$. So

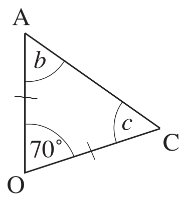
$$\begin{aligned} 90° + 20° + a &= 180° \\ a &= 180° - 110° \\ &= 70° \end{aligned}$$



Then consider the triangle OAC. As OA and OC are both radii of the circle, it is an isosceles triangle with $b = c$.

So

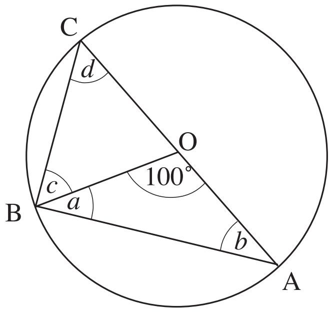
$$\begin{aligned} 2b + 70° &= 180° \\ 2b &= 110° \\ b &= 55° \end{aligned}$$



and $c = 55°$.

Worked Example 3

Find the angles marked in the diagram, where O is the centre of the circle.



Solution

First consider the triangle OAB.

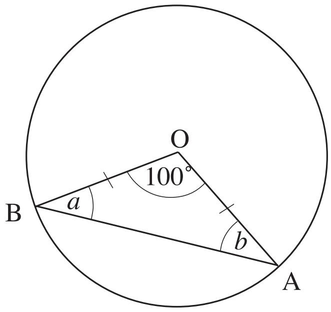
As the sides OA and OB are both radii, the triangle must be isosceles with $a = b$.

So $$a + b + 100° = 180°$$

but as $a = b$,

$$2a + 100° = 180°$$
$$2a = 80°$$
$$a = 40°$$

and $b = 40°$.



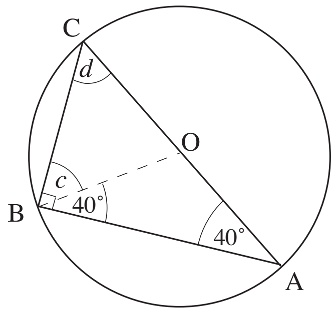
Now consider the triangle ABC.

As the line AC is a diameter of the circle, the angle ABC must be $90°$.

So
$$a + c = 90°$$
or
$$40° + c = 90°$$
$$c = 50°$$

The angles in the triangle ABC must total $180°$, so

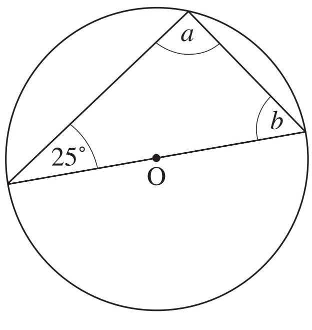
$$40° + 90° + d = 180°$$
$$d = 50°$$



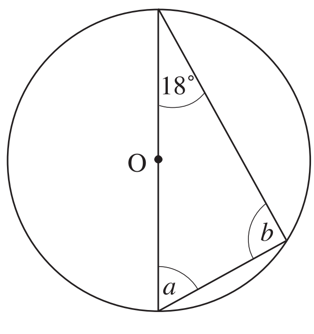
Exercises

1. Find the angles marked with a letter in each of the following diagrams. In each case the centre of the circle is marked O.

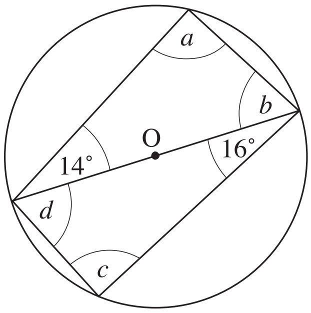
(a)



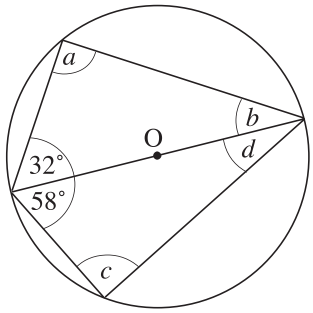
(b)



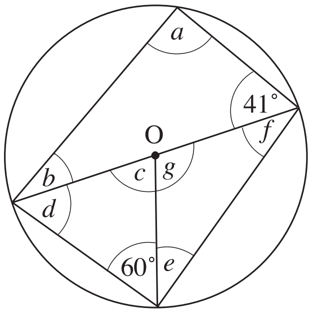
(c)



(d)



(e)



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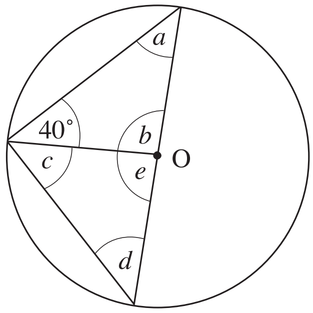
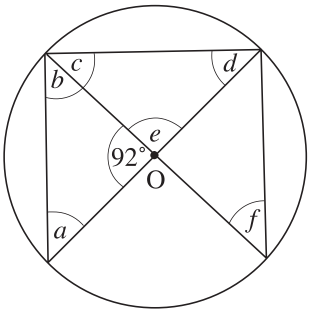
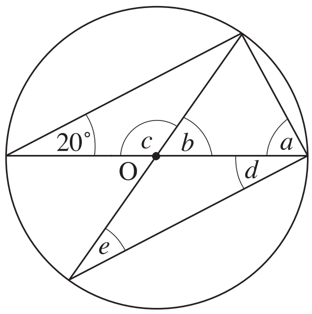


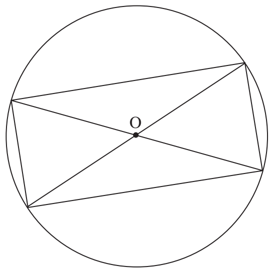
(g)



(h)

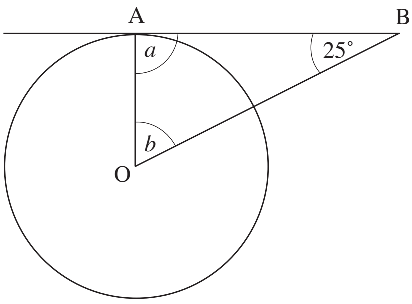


2. Copy the diagram opposite, and mark every right angle, if O is the centre of the circle.



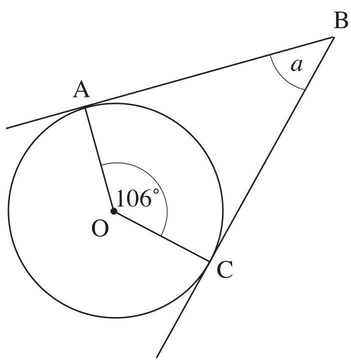
3. Find the angles marked with letters in each diagram below, if O is the centre of the circle.

(a)



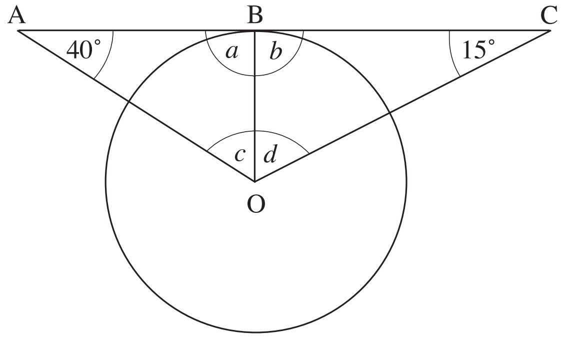
AB is a tangent

(b)



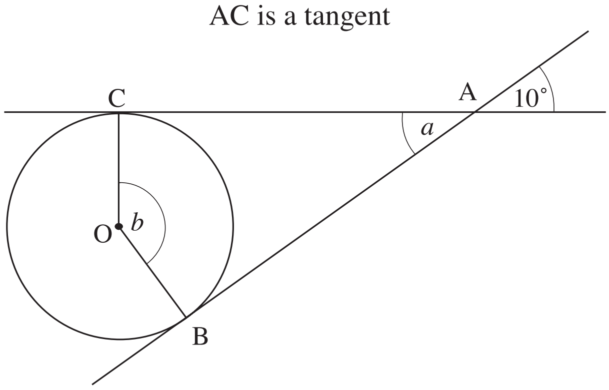
AB and BC are tangents

(c)



AC is a tangent

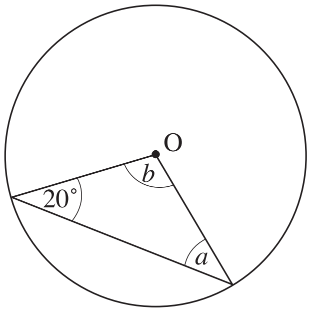
(d)



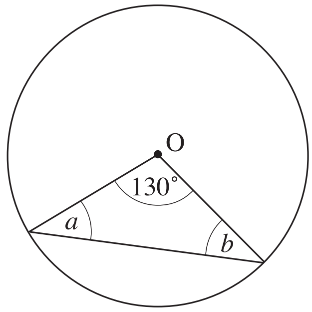
AB and AC are tangents

4. Find the angles marked with letters in each of the following diagrams, if O is the centre of the circle.

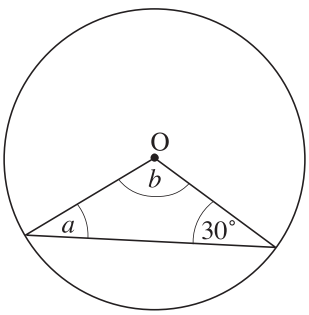
(a)



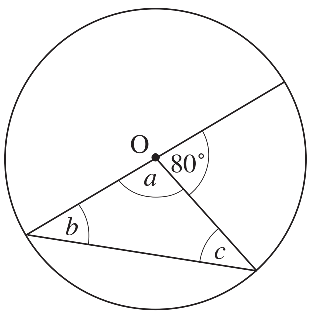
(b)



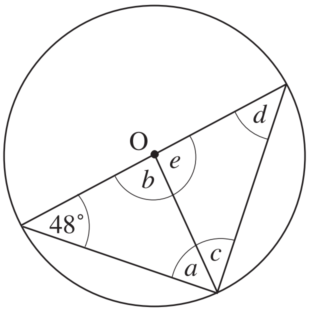
(c)



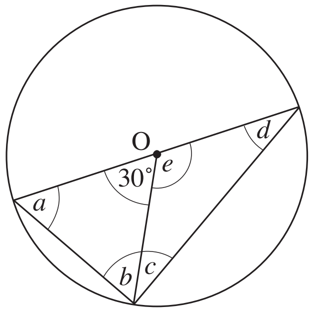
(d)



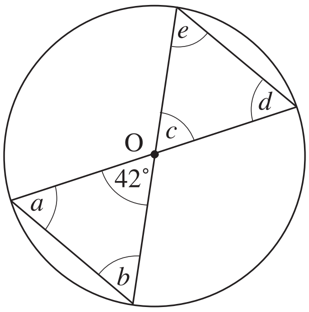
(e)



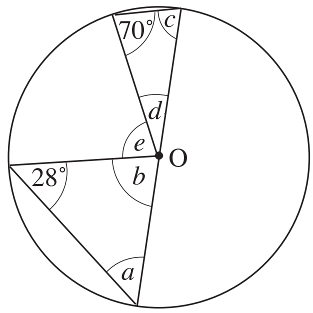
(f)



(g)

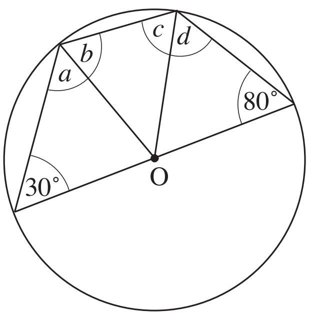


(h)

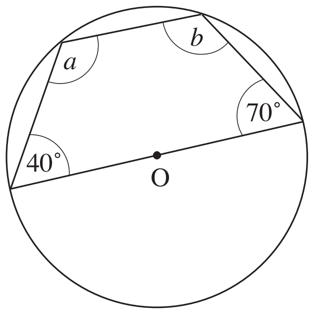


5. Find each of the marked angles if O is the centre of the circle.

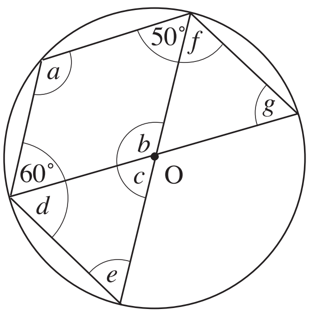
(a)



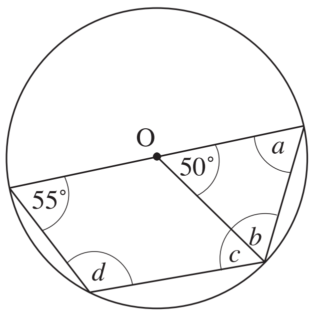
(b)



(c)

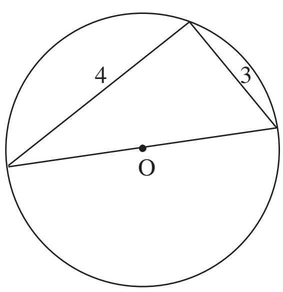


(d)

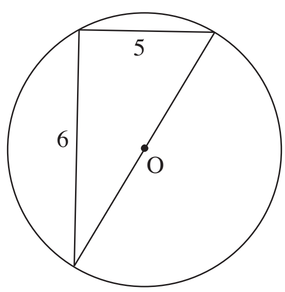


6. Find the diameter of each circle below, if O is the centre of the circle.

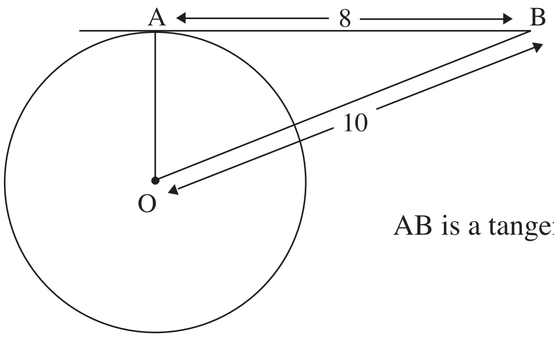
(a)



(b)

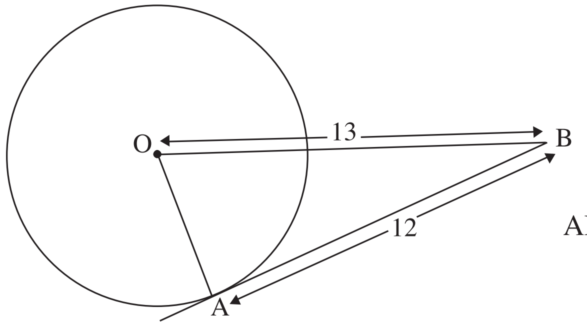


(c)



AB is a tangent

(d)

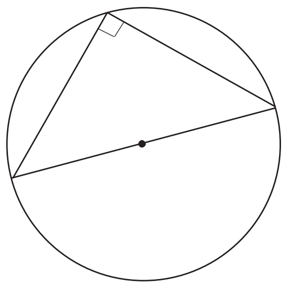


AB is a tangent

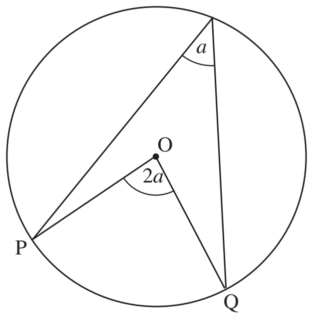
32.3 Angles and Circles 2

There are a number of important geometric results based on angles in circles. (The first you have met already.)

Any angle subtended at the circumference from a diameter is a right angle.



The angle subtended by an arc, PQ, at the centre is twice the angle subtended at the circumference.



Proof

OP = OC (equal radii), so

angle CPO = angle PCO (= x, say).

Similarly,

angle CQO = angle QCO (= y, say).

Now, extending the line CO to D, say, note that

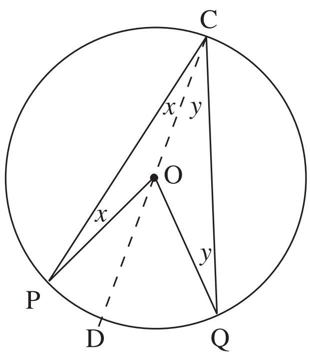
angle POD = $x + x$

= $2x$

and, similarly,

angle QOD = $y + y$

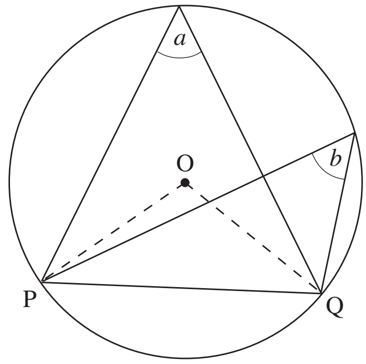
= $2y$



Hence,

$$\text{angle POQ} = 2x + 2y$$
$$= 2(x + y)$$
$$= 2 \times \text{angle PCQ}$$

as required.

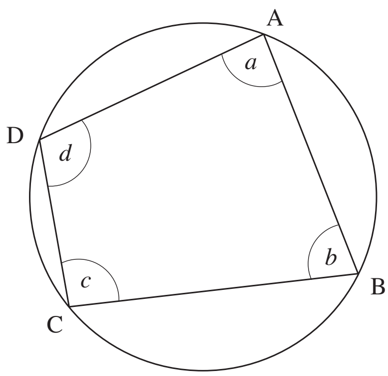


Angles subtended at the circumference by a chord (on the same side of the chord) are equal;
that is, in the diagram $a = b$.

Proof

The angle at the centre is $2a$ or $2b$ (according to the first result).

Thus $2a = 2b$ or $a = b$, as required.



In *cyclic quadrilaterals* (quadrilaterals where all 4 vertices lie on a circle), opposite angles sum to 180° *; that is*

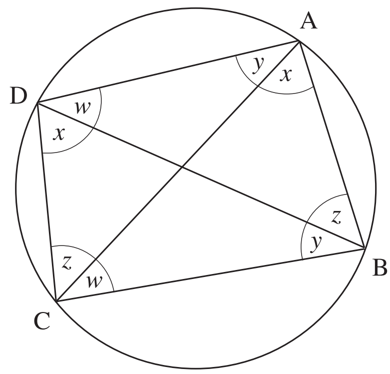
$$a + c = 180°$$
and
$$b + d = 180°$$

Proof

Construct the diagonals AC and BD, as below.

Then label the angles subtended by AB as w; that is

angle ADB = angle ACB (= w)



Similarly for the other chords, the angles being marked x, y and z as shown.

Now, in triangle ABD, the sum of the angles is 180°, so

$$w + z + (x + y) = 180°$$

You can rearrange this as

$$(x + w) + (y + z) = 180°$$

which shows that

angle CDA + angle CBA = 180°

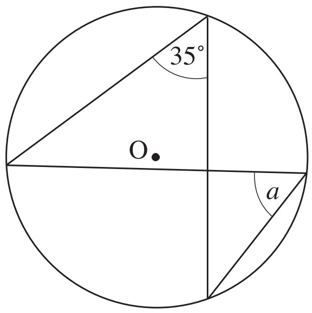
proving one of the results.

The other result follows in a similar way.

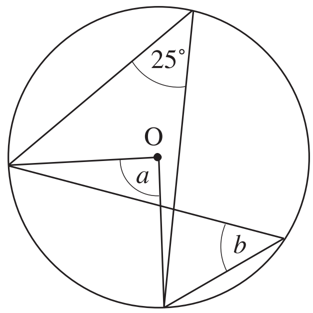
Worked Example 1

Find the angles marked in the diagrams. In each case O is the centre of the circle.

(a)



(b)



Solution

(a) As both angles are drawn on the same chord, the angles are equal, so

$$a = 35°$$

(b) Angle *b* and the $25°$ angle are drawn on the same chord, so

$$b = 25°$$

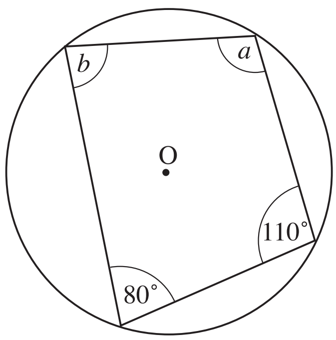
Angle *a* is drawn at the centre O on the same chord as the $25°$ angle, so

$$a = 2 \times 25°$$
$$= 50°$$

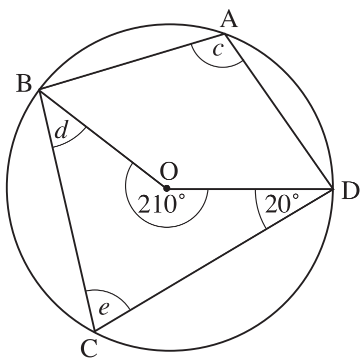
Worked Example 2

Find the angles marked in the diagrams. O is the centre of the circle.

(a)



(b)



Solution

(a) Opposite angles in a cyclic quadrilateral add up to $180°$. So

$$a + 80° = 180°$$
$$a = 100°$$

and

$$b + 110° = 180°$$
$$b = 70°$$

(b) Consider the angles c and $210°$. Since the angle at the centre is double the angle in a segment drawn on the same arc,

$$2c = 210°$$
$$c = 105°$$

Angles c and e add up to $180°$ because they are opposite angles in a cyclic quadrilateral.

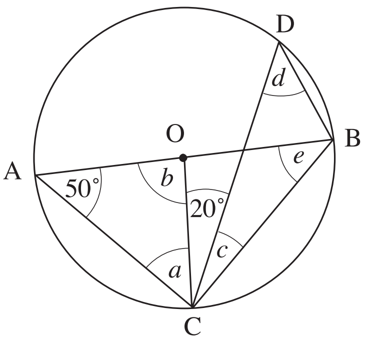
$$c + e = 180°$$
$$105° + e = 180°$$
$$e = 180° - 105°$$
$$= 75°$$

Consider the quadrilateral BODC. The four angles in any quadrilateral add up to $360°$. So

$$d + e + 210° + 20° = 360°$$
$$d = 360° - 210° - 20° - e$$
$$= 130° - c$$
$$= 130° - 75°$$
$$= 55°$$

Worked Example 3

In the diagram the line AB is a diameter and O is the centre of the circle. Find the angles marked.



Solution

Consider triangle OAC. Since OA and OC are radii, triangle OAC is isosceles. So

$$a = 50°$$

The angles in a triangle add to $180°$, so for triangle OAC,

$$a + b + 50° = 180°$$
$$b = 180° - 50° - a$$
$$= 80°$$

Since AB is a diameter of the circle, the angle ACB is a right angle, so

$$a + 20° + c = 90°$$
$$c = 90° - 20° - a$$
$$= 20°$$

Angle d and angle OAC are angles in the same segment, so

$$d = \text{angle OAC}$$
$$= 50°$$

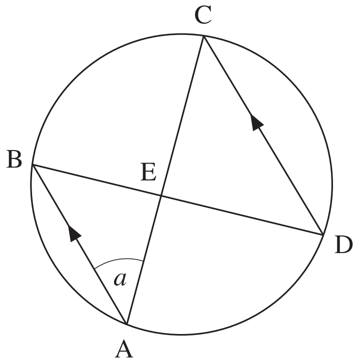
Angle e is drawn on the same arc as the angle at the centre, AOC, so

$$b = 2e$$

$$e = \frac{1}{2}b$$

$$= 40°$$

Worked Example 4



In the diagram the chords AB and CD are parallel.
Prove that the triangles ABE and DEC are isosceles.

Solution

Angles a and BDC are angles in the same segment, so

angle BDC = a

Since AB and DC are parallel, angles a and ACD are equal alternate angles,

angle ACD = a = angle BDC

Hence in triangle DEC, the base angles at C and D are equal, so the triangle is isosceles.

The angle at B, angle ABD, equals the angle at C, angle ACD, because they are angles in the same segment:

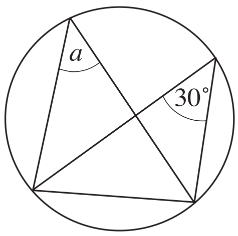
angle ABD = angle ACD = a

Hence triangle ABE is isosceles, since the angles at A and B are equal.

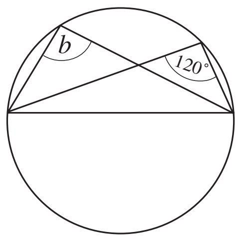
Exercises

1. Find all the angles marked with a letter in each of the following diagrams.
   In each case the centre of the circle is marked O. *Give reasons for your answers*.

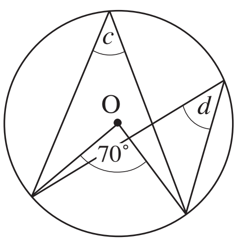
(a)



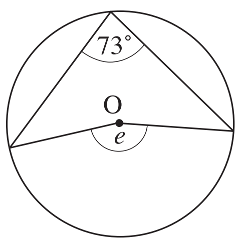
(b)



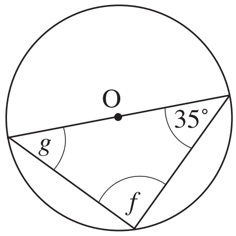
(c)



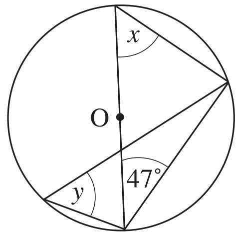
(d)



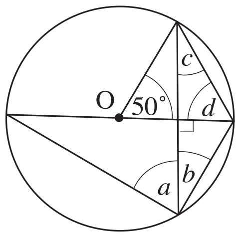
(e)



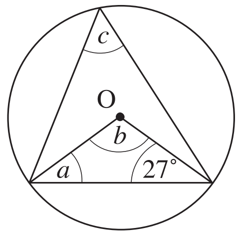
(f)



(g)



(h)

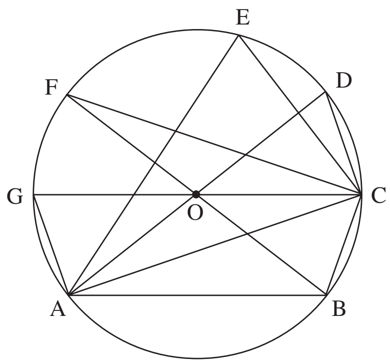


2. In the diagram, O is the centre of the circle and AOD, BOF and COG are diameters.

   (a) Identify the equal angles.

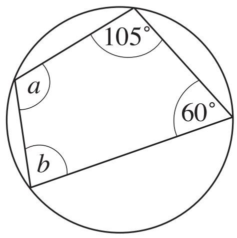
   (b) Identify the right angles.

   Give reasons for your answers.

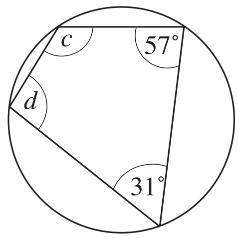


3. Find all of the angles marked with a letter in each of the following diagrams.
   Give reasons for your answers.

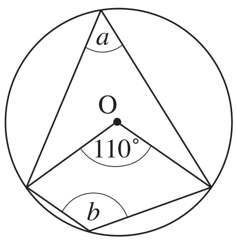
(a)



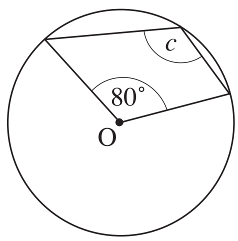
(b)



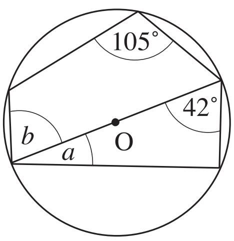
(c)



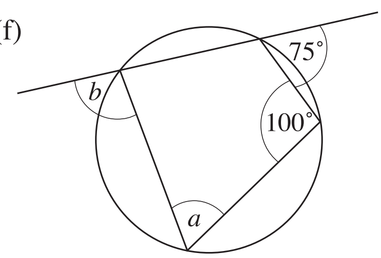
(d)



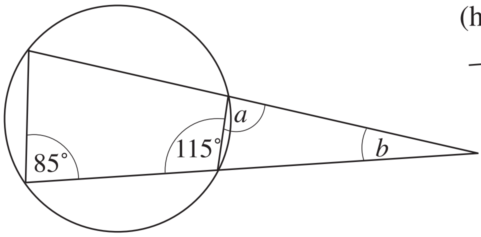
(e)



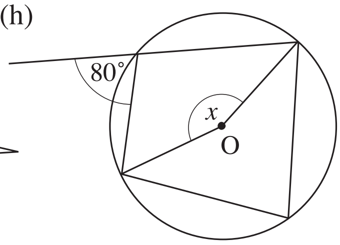
(f)



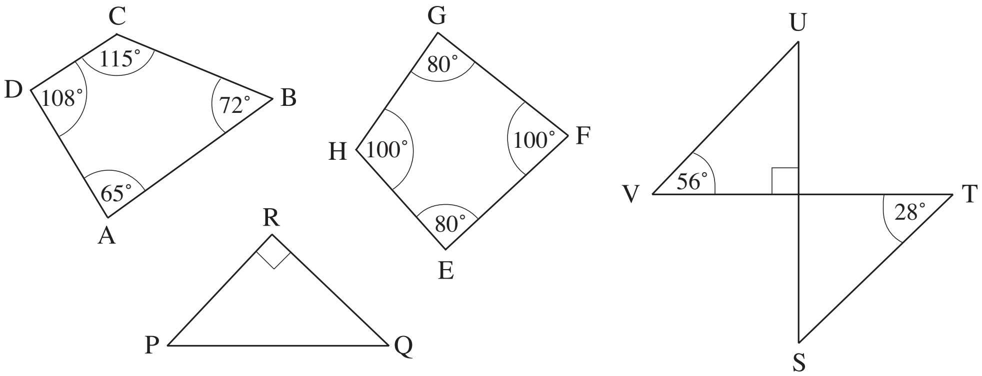
(g)



(h)

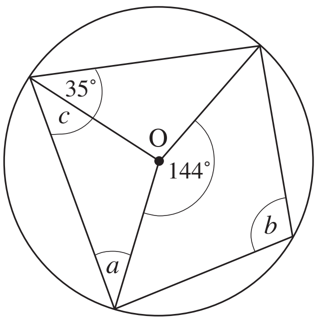


4. Which of the following points are concyclic points?

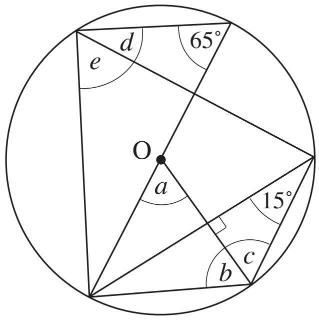


5. Find all the angles marked with a letter in the following diagrams. In each case, the point O is the centre of the circle.

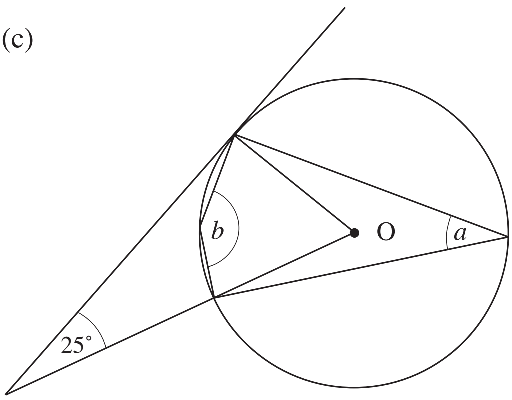
(a)



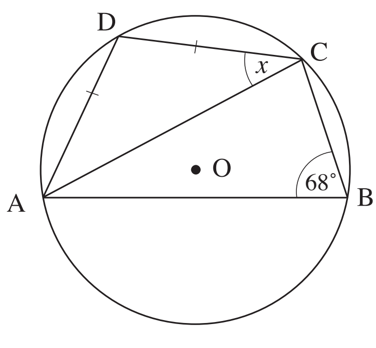
(b)



(c)



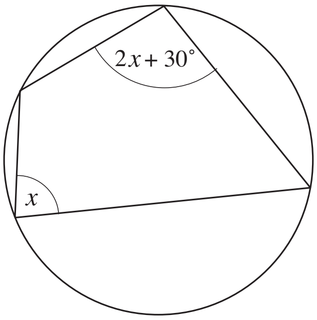
(d)



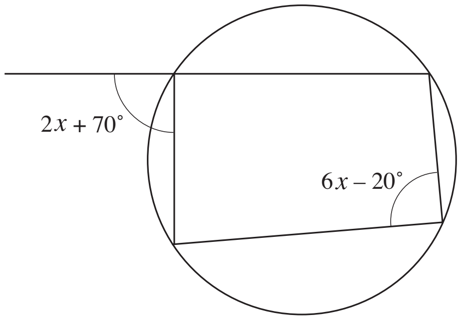
AD = DC

6. Find the value of x in each of the following diagrams.

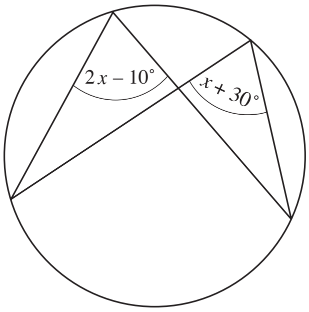
(a)



(b)

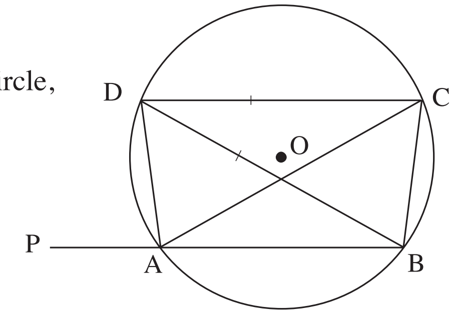


(c)



7. In the diagram, O is the centre of the circle, BD = DC and PAB is a straight line.

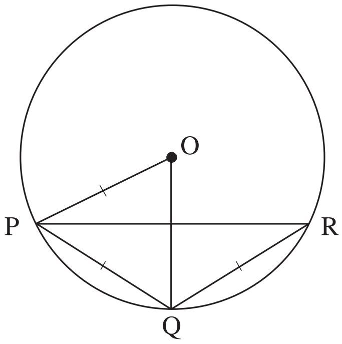
Prove that AD bisects the angle CAP.



8. In the diagram O is the centre of the circle,

$$OP = PQ = QR$$

Prove that OP and QR are parallel lines.

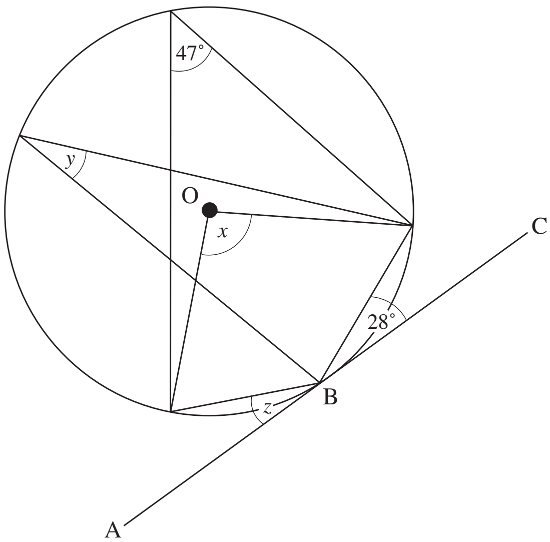


9. O is the centre of the circle.

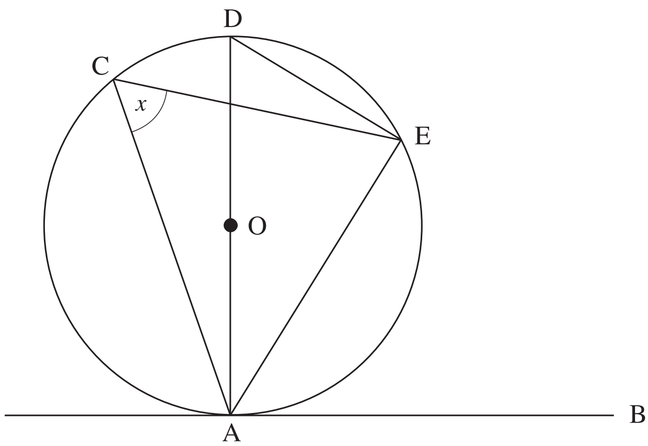
ABC is a tangent to the circle at B.

Not to scale

Work out the size of angles x, y and z.



10.



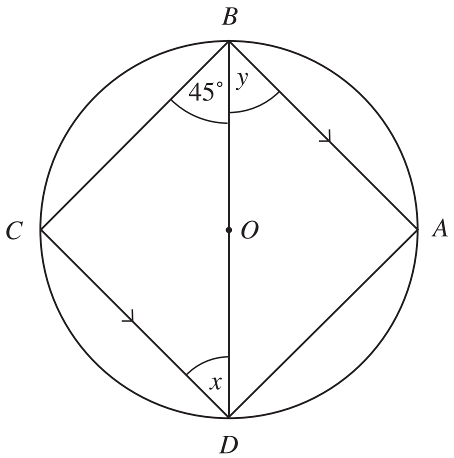
In the diagram, O is the centre of the circle, AD is a diameter and AB is a tangent.

Angle ACE = $x°$.

Find, in terms of x, the size of:

(a) angle ADE (b) angle DAE (c) angle EAB (d) angle AOE

11.



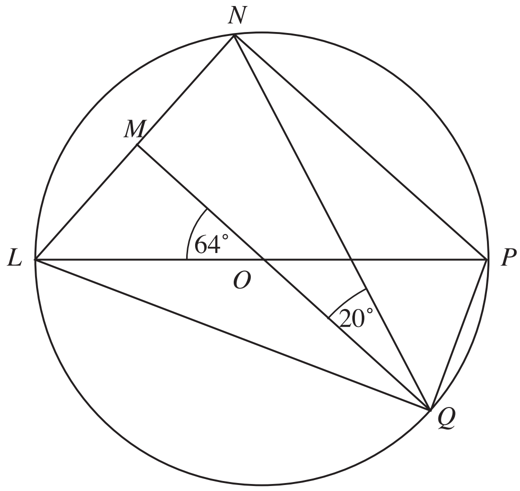
The diagram above, **not drawn to scale**, shows a circle, centre *O*. *BA* is parallel to *CD* and $C\hat{B}D = 45°$.

(a) Calculate, giving reasons, the values of x and y.

(b) Show that *ABCD* is a square, giving the reasons for your answer.

(CXC)

12.



The diagram shows a circle *LNPQ*, **not drawn to scale**, with centre *O*, angle $NQM = 20°$ and angle $MOL = 64°$.

Calculate, in degrees, giving reasons for your answers, the size of angles

(a) *OLQ*

(b) *NQP*

(c) *NLP*

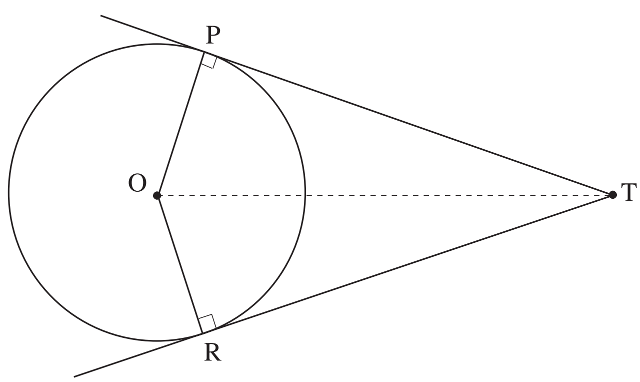
(d) *NPL*

(CXC)

32.4 Circles and Tangents

Some important results are stated below.

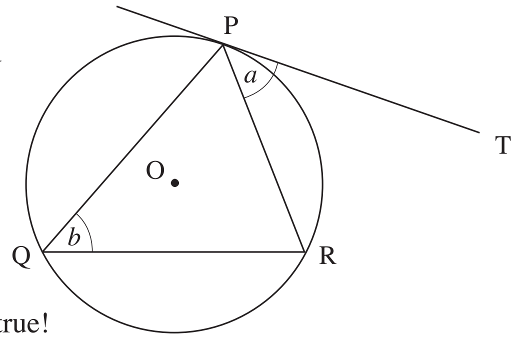
1. If two tangents are drawn from a point T to a circle with centre O, and P and R are the points of contact of the tangents with the circle, then, using symmetry,



(a) PT = RT

(b) Triangles TPO and TRO are congruent.

2. The angle between a tangent and a chord equals an angle at the circumference subtended by the same chord;
e.g. $a = b$ in the diagram.



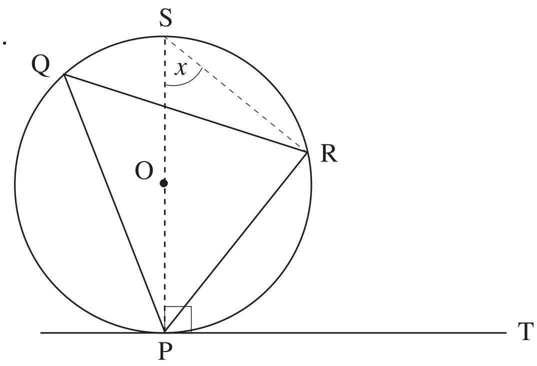
This is known as the

alternate segment theorem

and needs a proof, as it is not obviously true!

Proof

Construct the diameter POS, as shown.



We know that

$$\text{angle SRP} = 90°$$

since PS is a diameter.

Now

angle PSR = angle PQR = $x°$, say,

so

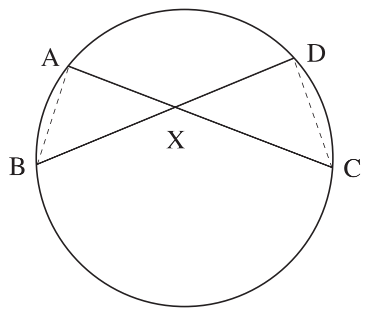
$$\text{angle SPR} = 180° - 90° - x$$
$$= 90° - x°$$

But

$$\text{angle RPT} = 90° - (\text{angle SPR})$$
$$= 90° - (90° - x°)$$
$$= x°$$
$$= \text{angle PQR}$$

and the result is proved.

3. For any two intersecting chords, as shown,



$$\boxed{AX \times CX = BX \times DX}$$

The proof is based on similar triangles.

Proof

In triangles AXB and DXC,

angle BAC = angle BDC (equal angles subtended by chord BC)

and

angle ABD = angle ACD (equal angles subtended by chord AD)

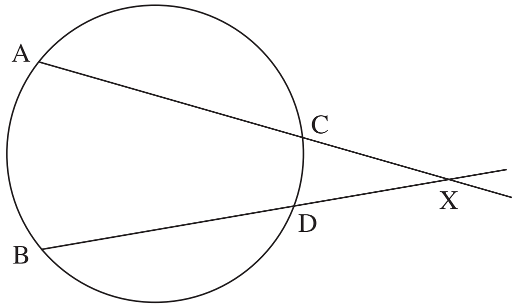
As AXB and DXC are similar,

$$\frac{AX}{BX} = \frac{DX}{CX} \quad \Rightarrow \quad AX.CX = BX.DX$$

as required.

This result will still be true even when the chords intersect *outside* the circle, as illustrated opposite.

How can this be proved?



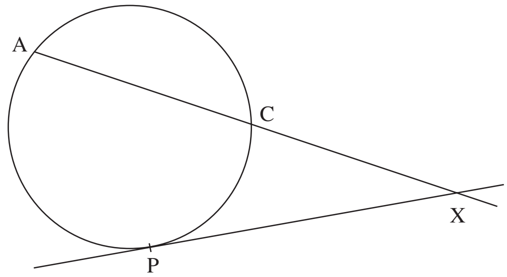
When the chord BD becomes a tangent,

and B and D coincide at the point P, then

$$AX \times CX = PX \times PX$$

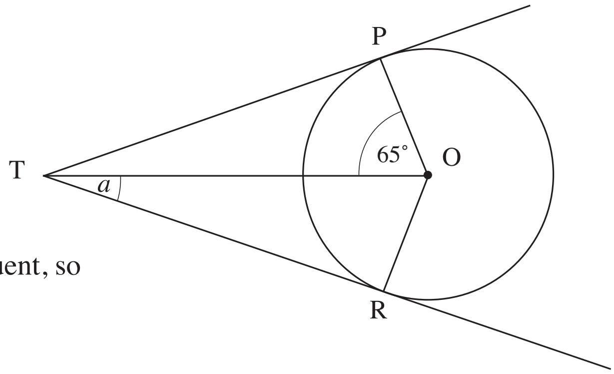
or

$$\boxed{AX \times CX = PX^2}$$



Worked Example 1

Find the angle a in the diagram.



Solution

The triangles TOR and TOP are congruent, so

$$\text{angle TOR} = 65°$$

Since TR is a tangent to the circle and OR is a radius,

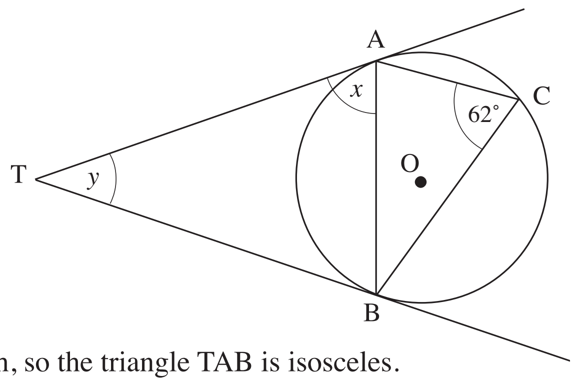
$$\text{angle TRO} = 90°.$$

Hence

$$a = 180° - 90° - 65°$$
$$= 25°$$

Worked Example 2

Find the angles x and y in the diagram.



Solution

The alternate angle segment theorem gives

$$x = 62°$$

The tangents TA and TB are equal in length, so the triangle TAB is isosceles.
So

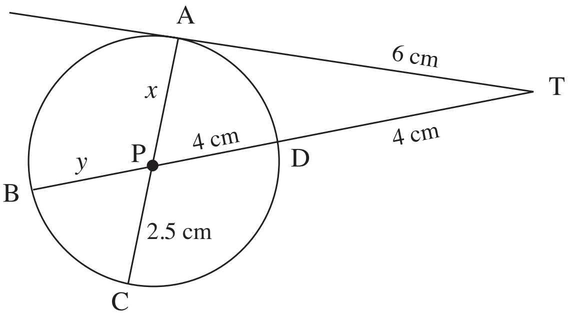
$$\text{angle ABT} = x = 62°$$

Hence

$$y + 62° + 62° = 180° \quad \text{(the angles in triangle TAB add up to } 180°\text{)}$$
$$y = 56°$$

Worked Example 3

Find the unknown lengths in the diagram.



Solution

Since AT is a tangent,

$$AT^2 = BT.DT$$
$$36 = BT \times 4$$
$$BT = 9$$

Hence

$$y + 8 = BT = 9$$
$$y = 1 \text{ cm}$$

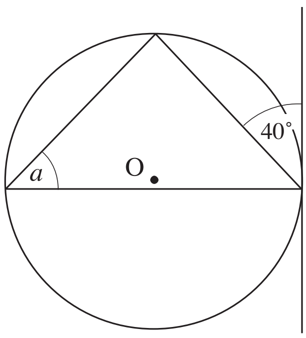
AC and BD are intersecting chords, so

$$AP.PC = BP.PD$$
$$2.5x = 1 \times 4$$
$$x = \frac{4}{2.5}$$
$$= 1.6 \text{ cm}$$

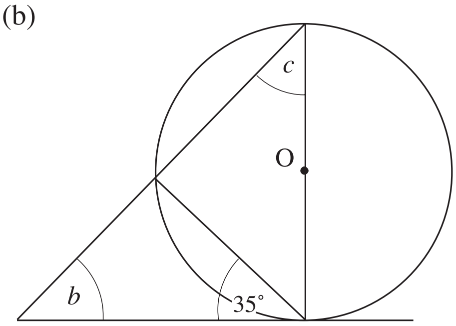
Exercises

1. Find the angles marked in the diagrams. In each case O is the centre of the circle.

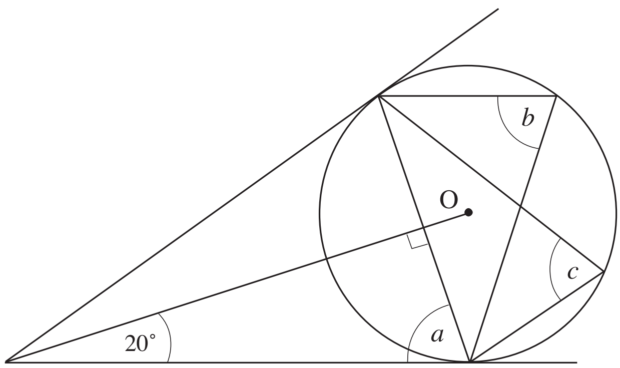
(a)



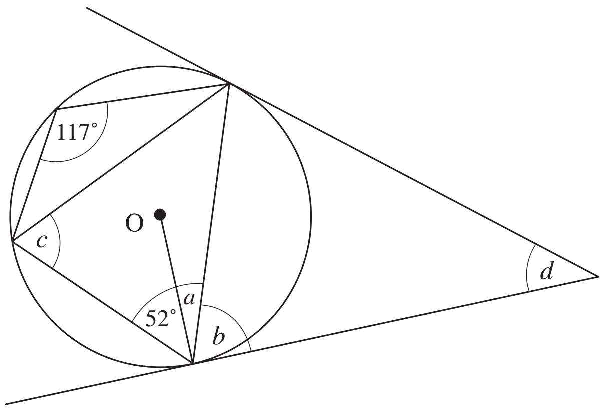
(b)



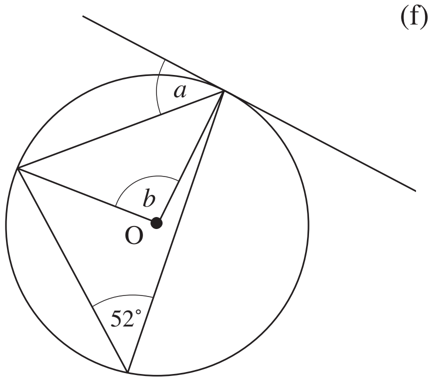
(c)



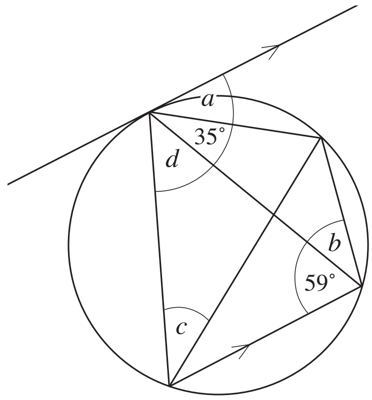
(d)



(e)

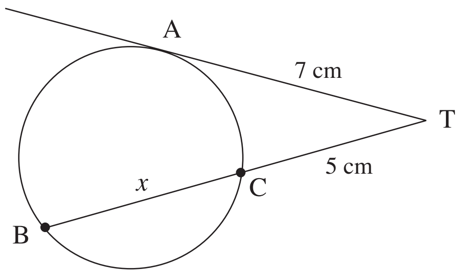


(f)

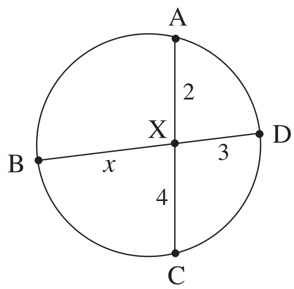


2. Find the unknown lengths in the following diagrams.

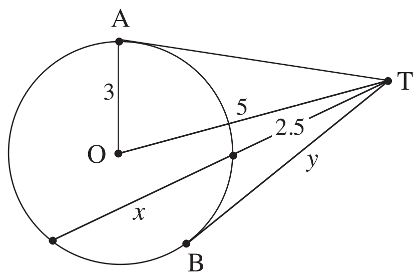
(a)



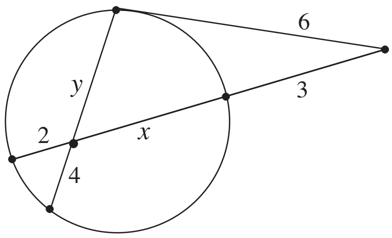
(b)



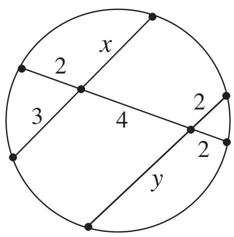
(c)



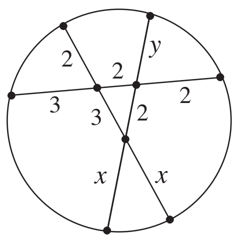
(d)



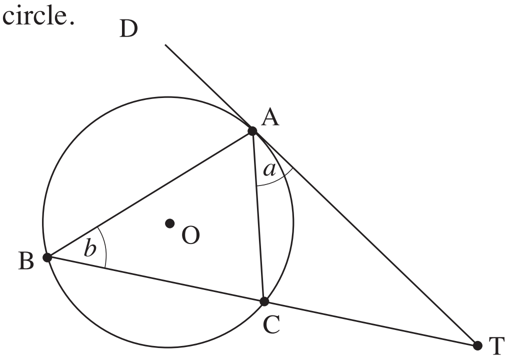
(e)



(f)



3. In the diagram, TAD is a tangent to the circle.



(a) Prove that $a = b$.

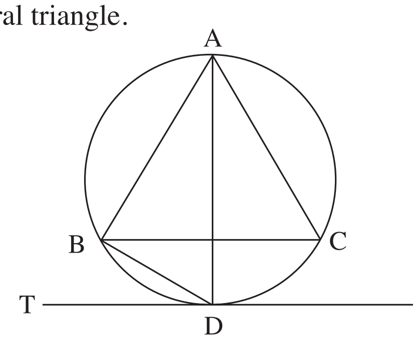
(b) Show that triangles BTA and ACT are similar triangles.

(c) If

$$BC = 5 \text{ cm}$$
$$CT = 4 \text{ cm}$$

calculate the length of the tangent AT.

4. The triangle ABC in the diagram is an equilateral triangle.

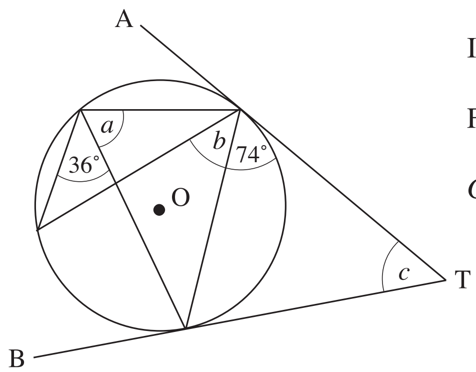


The line AD bisects angle BAC.

(a) Prove that the line AD is a diameter of the circle.

(b) Hence find the angle BDT, where DT is a tangent to the circle.

5.

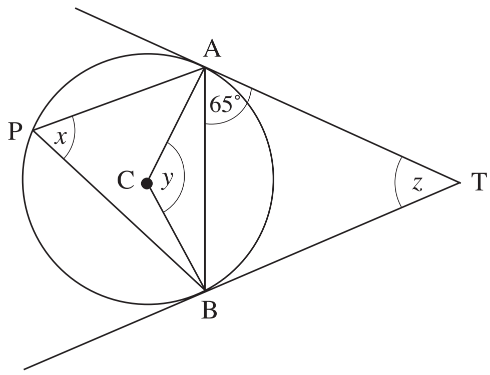


In the diagram, TA and TB are tangents.

Find the angles a, b and c.

Give reasons for your answers.

6. AT and BT are tangents to the circle, centre C.



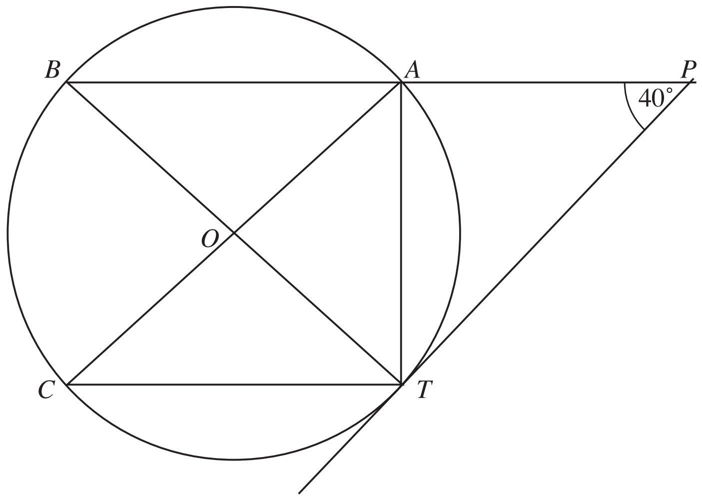
P is a point on the circumference, as shown.

$$\text{Angle BAT} = 65°$$

Calculate the size of

(a) x (b) y (c) z

7.



In the diagram above, **not drawn to scale**, *ABCT* is a circle. *AC* and *BT* are diameters. *TP,* the tangent at *T,* meets *BA* produced at *P,* so that $\angle APT = 40°$.

Calculate, **giving reasons for all statements**, the size of

(a) $\angle BTP$

(b) $\angle BAT$

(c) $\angle ABT$

(d) $\angle ACT$

(CXC)