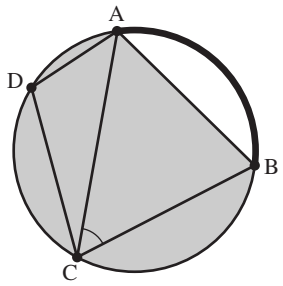
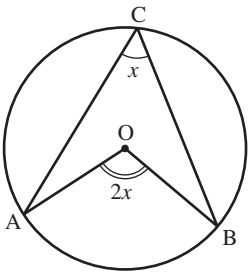
Circle theorems

There are four theorems on angles in circles that you should know. First you need to learn some words.



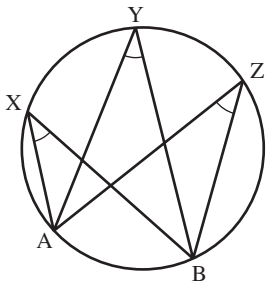
- The straight line AB is a **chord**.
- The curved line AB (in bold) is an **arc**.
- The chord AB divides the circle into two **segments** – the major segment (shaded) and the minor segment (unshaded).
- $A\hat{C}B$ is the angle **subtended** by AB at C.
- ABCD is a **cyclic quadrilateral**.

Theorem 1
The angle subtended at the centre of a circle is twice the angle subtended at the circumference.



$A\hat{O}B = 2 \times A\hat{C}B$

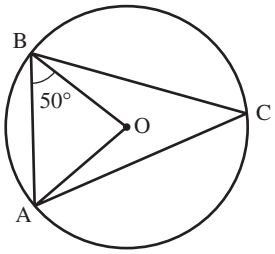
Theorem 2
Angles subtended by an arc in the same segment of a circle are equal.



$A\hat{X}B = A\hat{Y}B = A\hat{Z}B$

The proof of this theorem is given in the section on proof.

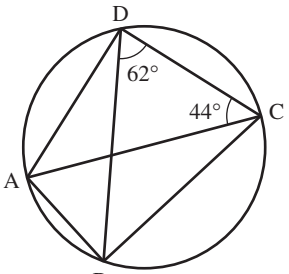
(a) Given $A\hat{B}O = 50°$, find $B\hat{C}A$.



Triangle OBA is isosceles (OA = OB).

$\therefore$ $O\hat{A}B = 50°$
$\therefore$ $B\hat{O}A = 80°$ (angle sum of a triangle)
$\therefore$ $B\hat{C}A = 40°$ (angle at the centre)

(b) Given $B\hat{D}C = 62°$ and $D\hat{C}A = 44°$ find $B\hat{A}C$ and $A\hat{B}D$.

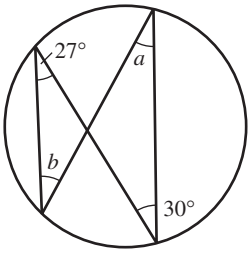


$B\hat{D}C = B\hat{A}C$ (both subtended by arc BC)
$\therefore$ $B\hat{A}C = 62°$
$D\hat{C}A = A\hat{B}D$ (both subtended by arc DA)
$\therefore$ $A\hat{B}D = 44°$

Exercise 1

Find the angles marked with letters. A line passes through the centre only when point O is shown.

1.



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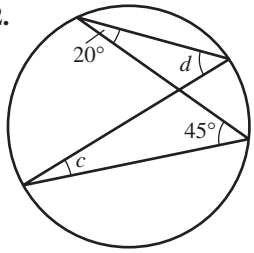

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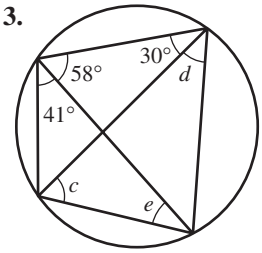

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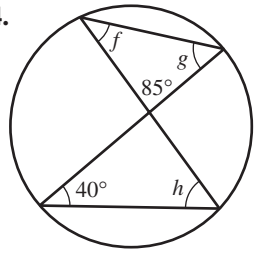

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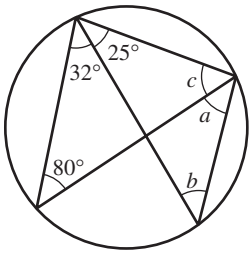

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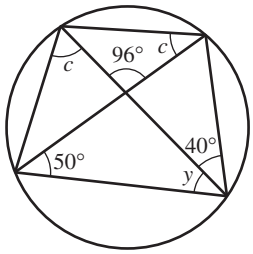

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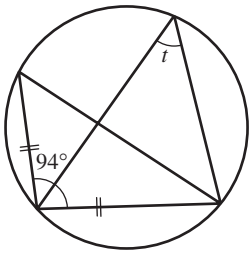

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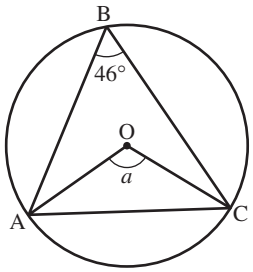

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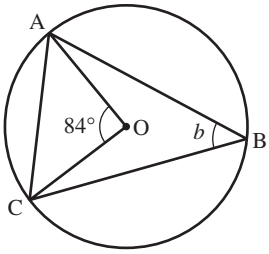

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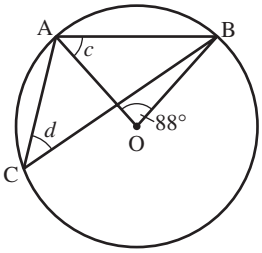

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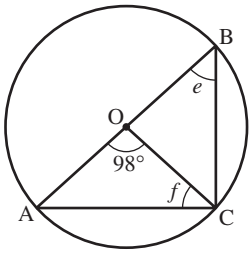

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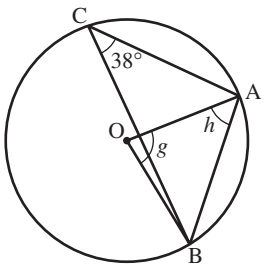

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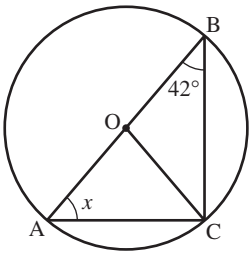

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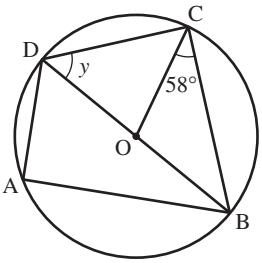

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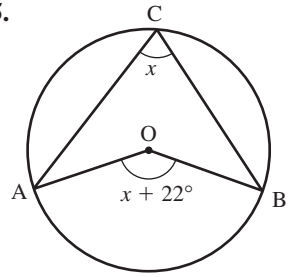

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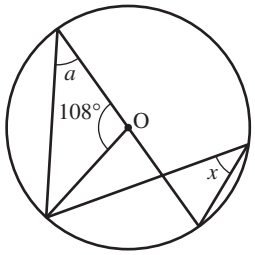
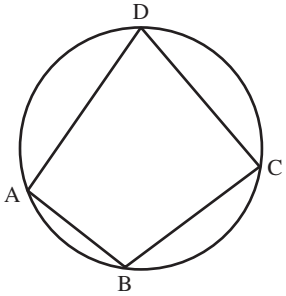


Theorem 3

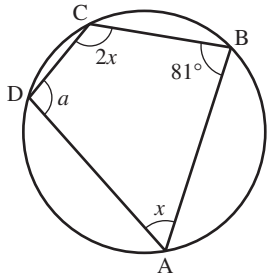
The opposite angles in a cyclic quadrilateral add up to 180° (the angles are supplementary).



$\widehat{A} + \widehat{C} = 180^\circ$
$\widehat{B} + \widehat{D} = 180^\circ$

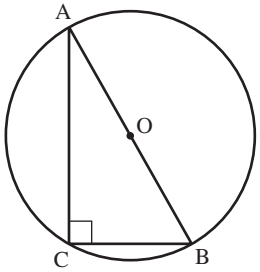
Find a and x.

$a = 180^\circ - 81^\circ$
(opposite angles of a cyclic quadrilateral)
$\therefore \quad a = 99^\circ$
$x + 2x = 180^\circ$
(opposite angles of a cyclic quadrilateral)
$3x = 180^\circ$
$\therefore \quad x = 60^\circ$



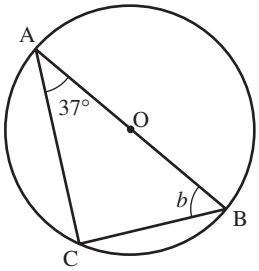
Theorem 4

The angle in a semicircle is a right angle.



In the diagram,
AB is a diameter.
$A\widehat{C}B = 90^\circ$.

Find b given that AOB is a diameter.



$A\widehat{C}B = 90^\circ$ (angle in a semicircle)
$\therefore \quad b = 180^\circ - (90 + 37)^\circ$
$= 53^\circ$

Exercise 2

Find the angles marked with a letter.

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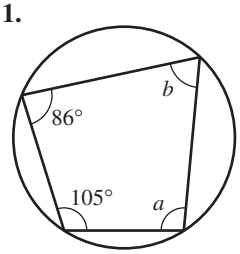

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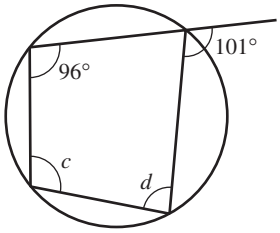

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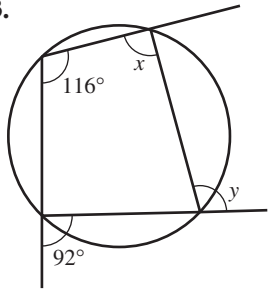

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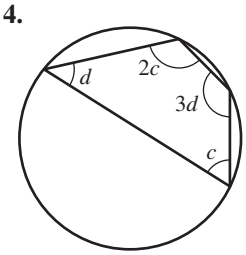

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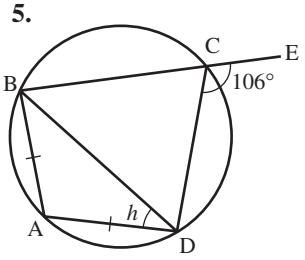

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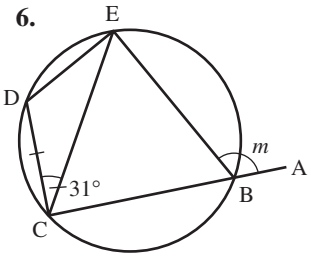

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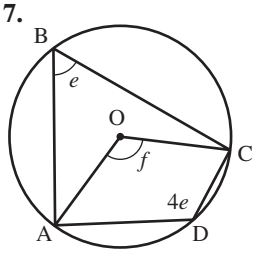

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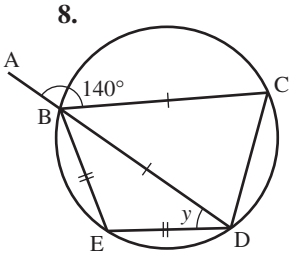

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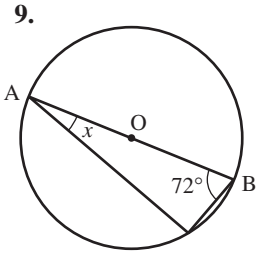

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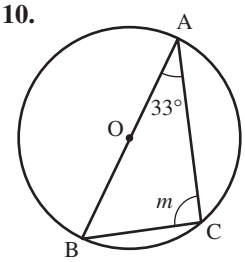

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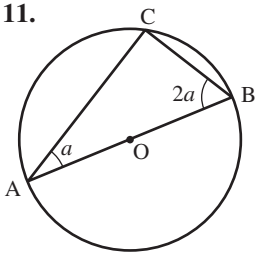

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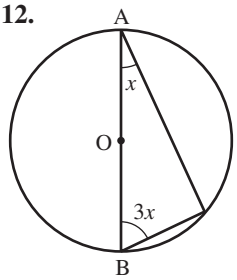

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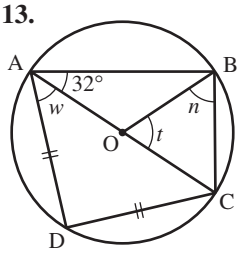

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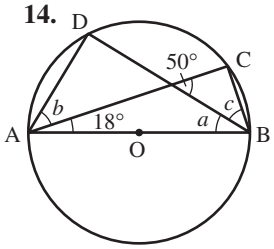

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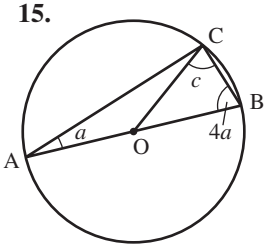

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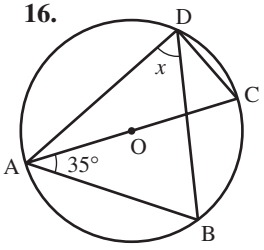

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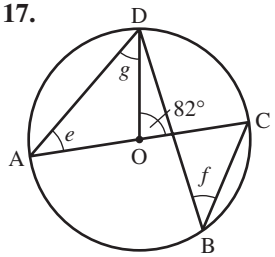

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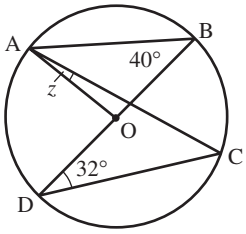

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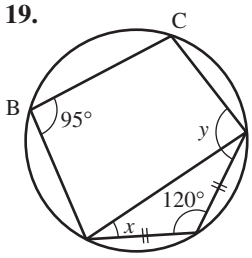

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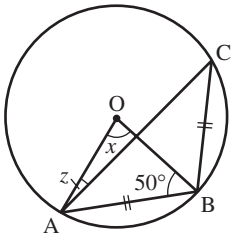


Moving averages

Here is a list of the number of children absent from a school over a 15-day period.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number absent	10	7	4	12	6	3	5	8	9	11	13	8	10	7	5

At the end of each day the mean number absent over *the last five days* is calculated.

So at the end of day 5, the mean over the last five days is $\frac{10+7+4+12+6}{5} = 7{\cdot}8$

At the end of day 6, the mean over the last five days is $\frac{7+4+12+6+3}{5} = 6{\cdot}4$

At the end of day 7, the mean over the last five days is $\frac{4+12+6+3+5}{5} = 6$

This is an example of a moving average.

Exercise 3

1. Traders on the stock market use moving averages as a guide to the performance of a company's share price. Here are the share prices, in pence, of a company over 30 days.

21	24	27	22	25	26	27	23	24	24
28	27	28	26	25	23	25	26	23	25
22	21	19	19	20	19	21	23	24	23

(a) What was the mean price over the first 10 days?
(b) What was the mean price over the 10 day period from day 2 to day 11?
(c) What was the mean price over the 10-day period from day 11 to day 20?

2. Here are the prices, in pence, of shares in 'Tiger Telecom' over a period of 20 days.

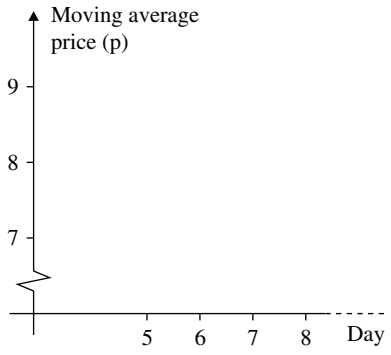
8	7	11	10	9	7	9	11	14	15
15	14	16	15	13	16	14	10	12	13

At the end of each day the mean price over the last 5 days is calculated.

So the mean price at the end of day 5 is $\frac{8+7+11+10+9}{5} = 9\text{p}$

The mean price on day 6 $= \frac{7+11+10+9+7}{5} = 8{\cdot}8\text{p}$

Work out the moving average price of the shares in this way up to day 20 and plot the results on a graph.



Answers

Exercise 1

1. $a = 27°$, $b = 30°$
2. $c = 20°$, $d = 45°$
3. $c = 58°$, $d = 41°$, $e = 30°$
4. $f = 40°$, $g = 55°$, $h = 55°$
5. $a = 32°$, $b = 80°$, $c = 43°$
6. $c = 34°$, $y = 34°$
7. $43°$
8. $92°$
9. $42°$
10. $c = 46°$, $d = 44°$
11. $e = 49°$, $f = 41°$
12. $g = 76°$, $h = 52°$
13. $48°$
14. $32°$
15. $22°$
16. $a = 36°$, $x = 36°$

Exercise 2

1. $a = 94°$, $b = 75°$
2. $c = 101°$, $d = 84°$
3. $x = 92°$, $y = 116°$
4. $c = 60°$, $d = 45°$
5. $37°$
6. $118°$
7. $e = 36°$, $f = 72°$
8. $35°$
9. $18°$
10. $90°$
11. $30°$
12. $22\frac{1}{2}°$
13. $n = 58°$, $t = 64°$, $w = 45°$
14. $a = 32°$, $b = 40°$, $c = 40°$
15. $a = 18°$, $c = 72°$
16. $55°$
17. $e = 41°$, $f = 41°$, $g = 41°$
18. $8°$
19. $x = 30°$, $y = 115°$
20. $x = 80°$, $z = 10°$

Exercise 3

1. (a) 24·3p (b) 25p (c) 25·6p
2. averages: 9, 8·8, 9·2, 9·2, 10, 11·2, 12·8, 13·8, 14·8, 15, 14·6, 14·8, 14·8, 13·6, 13, 13