

CXC Linear programming questions

- (a) A firm uses a combination of large and small boxes to package the items it produces.
- Large boxes can hold 8 items.
Small boxes can hold 3 items.
- The firm wishes to package 84 or less items using no more than 18 boxes.
- Let L represent the number of large boxes used and S , the number of small boxes used.
- (i) Write down two inequalities, other than $L > 0$ and $S > 0$ to represent the information above.
 - (ii) Using a scale of 2 cm to represent 5 units on both axes, draw the graphs to represent the inequalities in (i) above and shade the UNWANTED region.
 - (iii) A profit of \$32 is made on a large box and \$8 on a small box.
Write down an expression to represent the total profit made on the boxes.
 - (iv) From your graph, determine the number of large boxes, L , and the number of small boxes, S , which will yield the maximum profit.
 - (v) Calculate the maximum profit.

Total 15 marks

- (a) A farmer grows x acres of peas and y acres of tomatoes. He has 12 acres available to plant peas and tomatoes.
- (i) Write an inequality in x and y to satisfy this condition.
- The farmer must plant at least 2 acres of peas and at least 3 acres of tomatoes.
- (ii) Write TWO inequalities to satisfy these conditions.
- The number of acres of tomatoes planted must NOT be more than twice the number of acres of peas planted.
- (iii) Write an inequality in x and y to satisfy this condition.
 - (iv) Using a scale of 1 cm to represent one acre on each axis, draw graphs of the inequalities you have written at (i), (ii) and (iii) above.
 - (v) By shading the UNWANTED region, label as S the region which satisfies all four inequalities. **(11 marks)**
- (b) The farmer makes a profit of \$75 on each acre of peas and \$50 on each acre of tomatoes.
- (i) Write an expression in x and y for the total profit, P , which may be earned on the peas and tomatoes.
 - (ii) Use the graph to determine the number of acres of peas and tomatoes the farmer should plant in order to make the maximum profit.
 - (iii) State the maximum profit. **(4 marks)**

Total 15 marks

10. A vendor buys x kg of peanuts and y kg of cashew nuts.

- (a) (i) To get a good bargain, she must buy a minimum of 10 kg of peanuts and a minimum of 5 kg of cashew nuts.

Write TWO inequalities which satisfy these conditions.

- (ii) She buys no more than 60 kg of nuts. Peanuts cost \$4.00 per kg and cashew nuts cost \$8.00 per kg and she spends at least \$200.

Write TWO inequalities which satisfy these conditions.

(5 marks)

- (b) Using a scale of 2 cm to represent 10 kg on each axis, draw the graph of the **FOUR** inequalities in (a) (i) and (a) (ii).

On your graph, shade **ONLY** the region which satisfies **all four** inequalities.

(6 marks)

- (c) The profit on the sale of 1 kg of peanuts is \$2.00 and on 1 kg of cashew nuts is \$5.00.

- (i) Using your graph, determine the number of kilograms of each type of nut the vendor must sell in order to make the maximum profit.

- (ii) Calculate the maximum profit.

(4 marks)

Total 15 marks

- (b) A farmer supplies his neighbours with x pumpkins and y melons daily, using the following conditions:

First condition : $y \geq 3$

Second condition : $y \leq x$

Third condition : the total number of pumpkins and melons must not exceed 12.

- (i) Write an inequality to represent the **THIRD** condition. (1 mark)

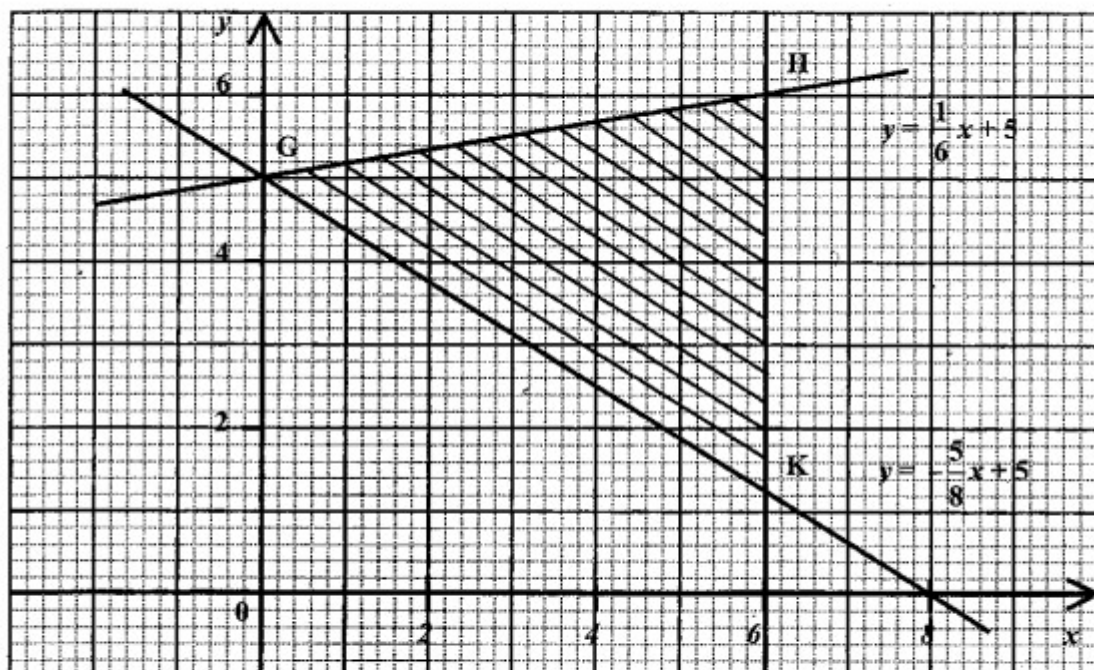
- (ii) Using a scale of **1 cm to represent one pumpkin** on the x -axis and **1 cm to represent one melon** on the y -axis, draw the graphs of the **THREE** lines associated with the **THREE** inequalities. (4 marks)

- (iii) Identify, by shading, the region which satisfies the **THREE** inequalities.

(1 mark)

- (iv) Determine, from your graph, the **minimum** values of x and y which satisfy the conditions. (2 marks)

- (c) The diagram below shows a triangular region bounded by the lines $y = \frac{1}{6}x + 5$, $y = -\frac{5}{8}x + 5$ and the line HK .



- (i) Write the equation of the line HK . (1 mark)
- (ii) Write the set of **three inequalities** which define the shaded region GHK . (3 marks)

10. The owner of a parking lot wishes to park x vans and y cars for persons attending a function. The lot provides parking space for no more than 60 vehicles.

- (i) Write an inequality to represent this information. **(2 marks)**

To get a good bargain, he must provide parking space for at least 10 cars.

- (ii) Write an inequality to represent this information. **(1 mark)**

The number of cars parked must be fewer than or equal to twice the number of vans parked.

- (iii) Write an inequality to represent this information. **(2 marks)**

(iv) (a) Using a scale of 2 cm to represent 10 vans on the x -axis and 2 cm to represent 10 cars on the y -axis, draw the graphs of the lines associated with the inequalities at (i), (ii) and (iii) above. **(5 marks)**

- (b) Identify by shading, the region which satisfies **all three** inequalities. **(1 mark)**

The parking fee for a van is \$6 and for a car is \$5.

(v) Write an expression in x and y for the total fees charged for parking x vans and y cars. **(1 mark)**

(vi) Using your graph write down the coordinates of the vertices of the shaded region. **(1 mark)**

- (vii) Calculate the maximum fees charged. **(2 marks)**

10. An answer sheet is provided for this question.

Pam visits the stationery store where she intends to buy x pens and y pencils.

- (a) Pam must buy at least 3 pens.

(i) Write an inequality to represent this information. (1 mark)

The TOTAL number of pens and pencils must NOT be more than 10.

(ii) Write an inequality to represent this information. (2 marks)

EACH pen costs \$5.00 and EACH pencil costs \$2.00. More information about the pens and pencils is represented by:

$$5x + 2y \leq 35$$

(iii) Write the information represented by this inequality as a sentence in your own words. (2 marks)

- (b) (i) On the answer sheet provided, draw the graph of the TWO inequalities obtained in (a) (i) and (a) (ii) above. (3 marks)

(ii) Write the coordinates of the vertices of the region that satisfies the four inequalities (including $y \geq 0$). (2 marks)

- (c) Pam sells the x pens and y pencils and makes a profit of \$1.50 on EACH pen and \$1.00 on EACH pencil.

(i) Write an expression in x and y to represent the profit Pam makes. (1 mark)

(ii) Calculate the maximum profit Pam makes. (2 marks)

(iii) If Pam buys 4 pens, show **on your graph** the maximum number of pencils she can buy. (2 marks)

Total 15 marks

10. A company manufactures gold and silver stars to be used as party decorations. The stars are placed in packets so that each packet contains x gold stars and y silver stars.

The conditions for packaging are given in the table below.

Condition	Inequality
(1) Each packet must have at least 20 gold stars	$x \geq 20$
(2) Each packet must have at least 15 silver stars	
(3) The total number of stars in each packet must not be more than 60.	
(4)	$x < 2y$

- (a) Write down the inequalities to represent conditions (2) and (3). (2 marks)
- (b) Describe, in words, the condition represented by the inequality $x < 2y$. (2 marks)
- (c) Using a scale of 2 cm to represent 10 units on both axes, draw the graphs of ALL FOUR inequalities represented in the table above. (7 marks)
- (d) Three packets of stars were selected for inspection. Their contents are shown below.

Packet	No. of gold stars (x)	No. of silver stars (y)
A	25	20
B	35	15
C	30	25

Plot the points A, B and C on your graph. Hence determine which of the three packets satisfy ALL the conditions. (4 marks)

Total 15 marks

10. (a) A school buys x balls and y bats.

The total number of balls and bats is no more than 30.

- (i) Write an inequality to represent this information. (2 marks)

The school budget allows no more than \$360 to be spent on balls and bats. The cost of a ball is \$6 and the cost of a bat is \$24.

- (ii) Write an inequality to represent this information. (2 marks)

- (b) (i) Using a scale of 2 cm on the x -axis to represent 10 balls and 2 cm on the y -axis to represent 5 bats, draw the graphs of the lines associated with the inequalities at (a) (i) and (ii) above. (5 marks)

- (ii) Shade the region which satisfies the two inequalities at (a) (i) and (ii) and the inequalities $x \geq 0$ and $y \geq 0$. (1 mark)

- (iii) Use your graph to write the coordinates of the vertices of the shaded region. (2 marks)

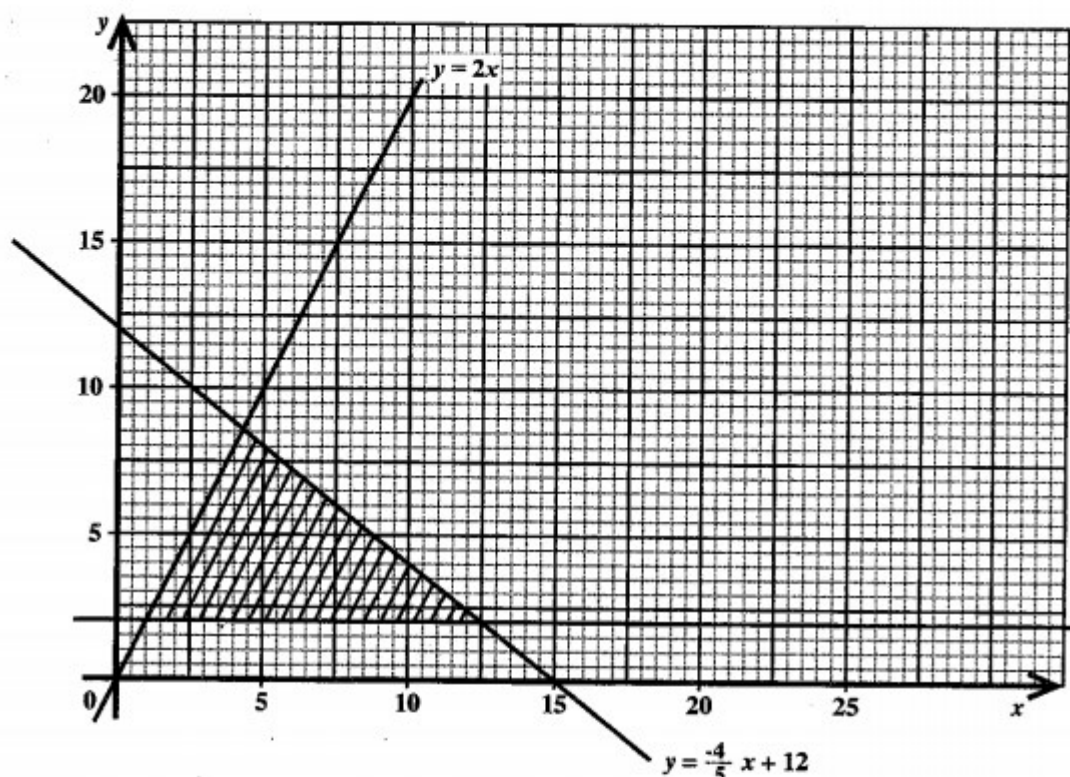
- (c) The balls and bats are sold to students. The school makes a profit of \$1 on each ball and \$3 on each bat. The equation $P = x + 3y$ represents the total profit that may be collected from the sale of these items.

- (i) Use the coordinates of the vertices given at (b) (iii) above to determine the profit for each of those combinations. (2 marks)

- (ii) Hence, state the maximum profit that may be made. (1 mark)

Total 15 marks

- (b) The shaded area in the diagram below shows the solution of a set of inequalities in x and y . The variable x represents the number of boys in a cricket club and y represents the number of girls in the cricket club.



Use the graph above to answer the questions which follow.

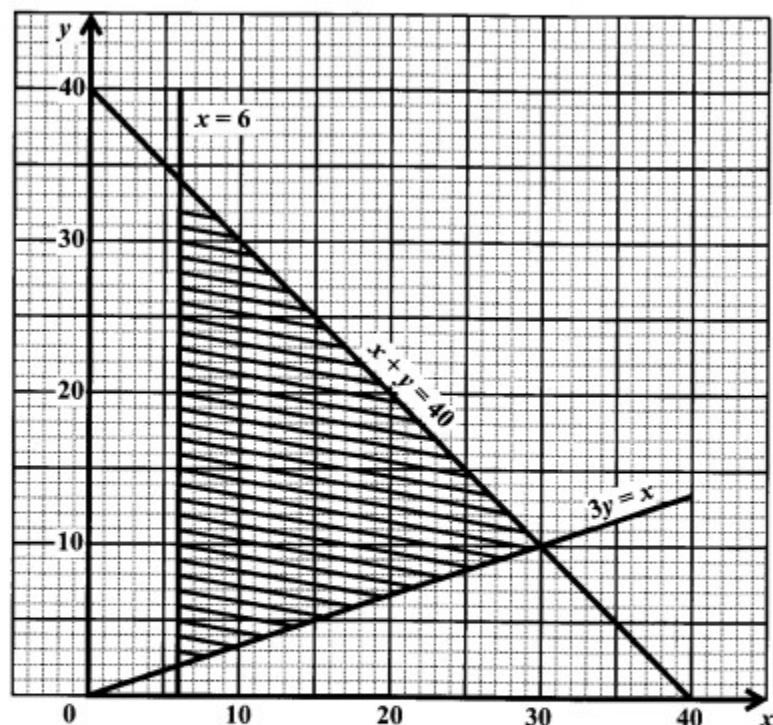
- (i) State, **using arguments based on the graph**, whether the cricket club can have as members:
 - a) 10 boys and 5 girls
 - b) 6 boys and 6 girls. (2 marks)
- (ii) Write down the set of **THREE** inequalities that define the shaded region. (4 marks)
- (iii) A company sells uniforms for the club and makes a profit of \$3.00 on a boy's uniform and \$5.00 on a girl's uniform.
 - a) Write an expression in x and y that represents the total profit made by the company on the sale of uniforms.
 - b) Calculate the **minimum** profit the company can make. (3 marks)

10. (a) The owner of a shop wishes to buy x guitars and y violins. To satisfy the demands of his customers, the number of violins must be less than or equal to the number of guitars.
- (i) Write an inequality to represent this information. (1 mark)
- The cost of one guitar is \$150 and the cost of one violin is \$300. He has \$4 500 to spend on the purchase of these instruments.
- (ii) Write an inequality to represent this information. (2 marks)
- To get a good bargain, the owner of the shop must buy at least 5 violins.
- (iii) Write an inequality to represent this information. (1 mark)
- (b) (i) Using a scale of **2 cm on the horizontal axis to represent 5 guitars**, and **2 cm on the vertical axis to represent 5 violins**, draw the graphs of the lines associated with the **THREE** inequalities written in (a) (i), (ii) and (iii) above. (4 marks)
- (ii) Shade the region on your graph that satisfies all **THREE** inequalities. (1 mark)
- (iii) State the coordinates of the vertices of the shaded region. (2 marks)
- (c) The owner of the shop sells the instruments to make a profit of \$60 on each guitar and \$100 on each violin.
- (i) Express the **TOTAL** profit in terms of x and y . (1 mark)
- (ii) Calculate the **maximum** profit. (3 marks)

10. (a) The manager of a pizza shop wishes to make x small pizzas and y large pizzas. His oven holds no more than 20 pizzas.
- (i) Write an inequality to represent the given condition. (2 marks)
- The ingredients for each small pizza cost \$15 and for each large pizza \$30. The manager plans to spend no more than \$450 on ingredients.
- (ii) Write an inequality to represent this condition. (2 marks)
- (b) (i) **Using a scale of 2 cm on the x -axis to represent 5 small pizzas and 2 cm on the y -axis to represent 5 large pizzas**, draw the graphs of the lines associated with the inequalities at (a) (i) and (a) (ii) above. (4 marks)
- (ii) Shade the region which is defined by ALL of the following combined:
- the inequalities written at (a) (i) and (a) (ii)
 - the inequalities $x \geq 0$ and $y \geq 0$
- (1 mark)
- (iii) Using your graph, state the coordinates of the vertices of the shaded region. (2 marks)
- (c) The pizza shop makes a profit of \$8 on the sale of EACH small pizza and \$20 on the sale of EACH large pizza. All the pizzas that were made were sold.
- (i) Write an expression in x and y for the TOTAL profit made on the sale of the pizzas. (1 mark)
- (ii) Use the coordinates of the vertices given at (b) (iii) to determine the MAXIMUM profit. (3 marks)

- (b) The diagram below shows the graphs of three lines and a shaded region defined by three inequalities associated with these lines.

The inequality associated with the line $3y = x$ is $3y \geq x$.



- (i) State the other TWO inequalities which define the shaded region. (2 marks)

The function $p = 4x + 3y$ satisfies the solution set represented by the closed triangular region.

- (ii) Identify the three pairs of (x, y) values for which p has a maximum or a minimum value. (3 marks)
- (iii) Which pair of (x, y) values makes p a maximum? **Justify your answer.** (4 marks)