FORM TP 2010015

TEST CODE **01234020**

JANUARY 2010

CARIBBEAN EXAMINATIONS COUNCIL

SECONDARY EDUCATION CERTIFICATE EXAMINATION

MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

05 JANUARY 2010 (a.m.)

INSTRUCTIONS TO CANDIDATES

1. Answer ALL questions in Section I, and ANY TWO in Section II.
2. Write your answers in the booklet provided.
3. All working must be clearly shown.
4. A list of formulae is provided on page 2 of this booklet.

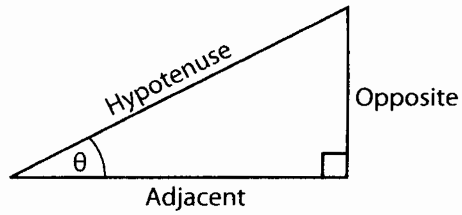
Examination Materials

Electronic calculator (non-programmable)
Geometry set
Mathematical tables (provided)
Graph paper (provided)

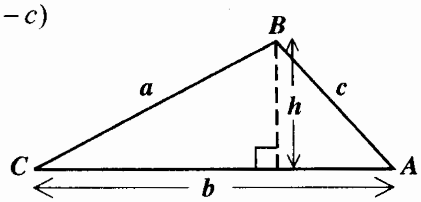
DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

LIST OF FORMULAE

Volume of a prism	$V = Ah$ where A is the area of a cross-section and h is the perpendicular length.
Volume of cylinder	$V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.
Volume of a right pyramid	$V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.
Circumference	$C = 2\pi r$ where r is the radius of the circle.
Area of a circle	$A = \pi r^2$ where r is the radius of the circle.
Area of trapezium	$A = \frac{1}{2}(a + b)\,h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.
Roots of quadratic equations	If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Trigonometric ratios	$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$, $\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$, $\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$



Area of triangle	Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height
	Area of $\Delta ABC = \frac{1}{2}ab\sin C$
	Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$
Sine rule	$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
Cosine rule	$a^2 = b^2 + c^2 - 2bc\cos A$



SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the exact value of

$$\frac{2.76}{0.8} + 8.7^2 .$$

(3 marks)

(b) In a certain company, a salesman is paid a fixed salary of \$3 140 per month plus an annual commission of 2% on the TOTAL value of cars sold for the year. If the salesman sold cars valued at \$720 000 in 2009, calculate

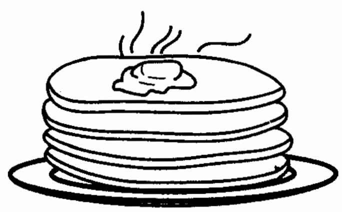
(i) his fixed salary for the year **(1 mark)**

(ii) the amount he received in commission for the year **(1 mark)**

(iii) his TOTAL income for the year. **(1 mark)**

(c) The ingredients for making pancakes are shown in the diagram below.

Ingredients for making 8 pancakes



2 cups pancake mix

$1\frac{1}{3}$ cups milk

(i) Ryan wishes to make 12 pancakes using the instructions given above. Calculate the number of cups of pancake mix he must use. **(2 marks)**

(ii) Neisha used 5 cups of milk to make pancakes using the same instructions. How many pancakes did she make? **(3 marks)**

Total 11 marks

2. (a) Given that $a = 6$, $b = -4$ and $c = 8$, calculate the value of

$$\frac{a^2 + b}{c - b}.$$

(3 marks)

(b) Simplify the expression:

(i) $3(x - y) + 4(x + 2y)$ (2 marks)

(ii) $\dfrac{4x^2 \times 3x^4}{6x^3}$ (3 marks)

(c) (i) Solve the inequality

$$x - 3 < 3x - 7.$$

(3 marks)

(ii) If x is an integer, determine the SMALLEST value of x that satisfies the inequality in (c) (i) above. (1 mark)

Total 12 marks

3. (a) T and E are subsets of a universal set, U, such that:

$U = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 \}$

$T = \{$ multiples of 3 $\}$

$E = \{$ even numbers $\}$

(i) Draw a Venn diagram to represent this information. (4 marks)

(ii) List the members of the set

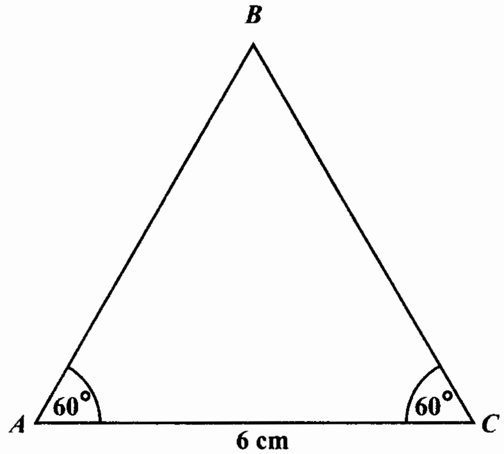
a) $T \cap E$ (1 mark)

b) $(T \cup E)'$. (1 mark)

(b) **Using a pencil, a ruler, and a pair of compasses only:**

(i) Construct, accurately, the triangle ABC shown below, where,

$AC = 6$ cm
$\angle ACB = 60°$
$\angle CAB = 60°$



(3 marks)

(ii) Complete the diagram to show the kite, $ABCD$, in which $AD = 5$ cm.
(2 marks)

(iii) Measure and state the size of $\angle DAC$. **(1 mark)**

Total 12 marks

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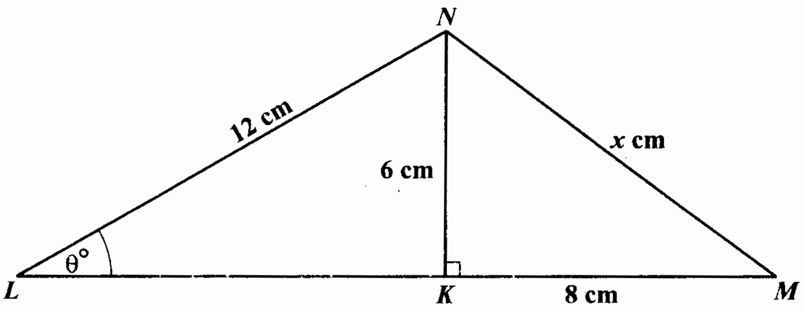
4. (a) The diagram below, **not drawn to scale**, shows a triangle LMN with $LN = 12$ cm, $NM = x$ cm and $\angle NLM = \theta$ °. The point K on LM is such that NK is perpendicular to LM, $NK = 6$ cm, and $KM = 8$ cm.

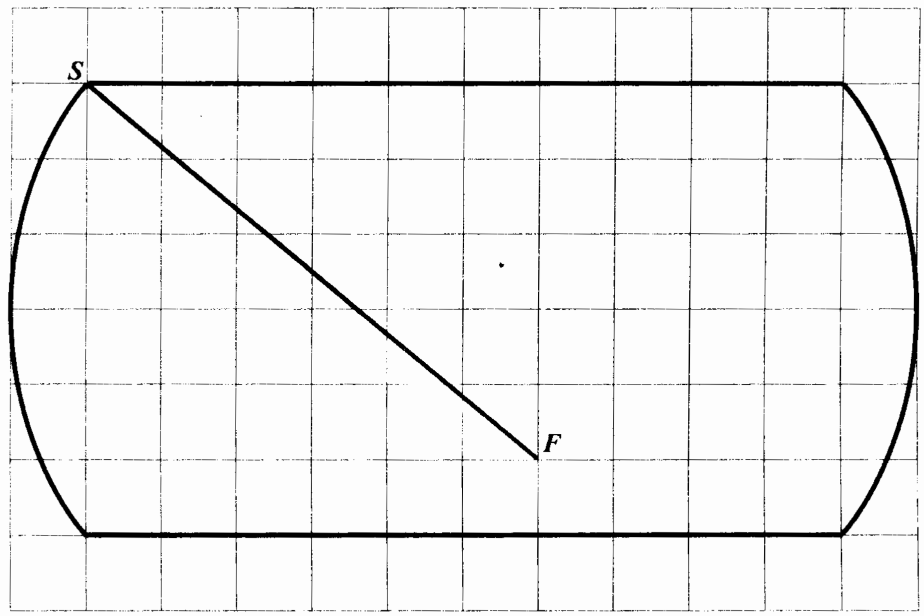


Calculate the value of

(i) x **(2 marks)**

(ii) θ. **(3 marks)**

(b) The diagram below shows a map of a playing field drawn on a grid of 1 cm squares. **The scale of the map is 1 : 1 250.**



(i) Measure and state, in centimetres, the distance from S to F on the map. **(1 mark)**

(ii) Calculate the distance, in metres, from S to F on the ACTUAL playing field. **(2 marks)**

(iii) Daniel ran the distance from S to F in 9.72 seconds. Calculate his average speed in

a) m/s

b) km/h

giving your answer correct to 3 significant figures. **(3 marks)**

Total 11 marks

5. (a) A straight line passes through the point $T(4,1)$ and has a gradient of $\frac{3}{5}$. Determine the equation of this line. **(3 marks)**

(b) (i) **Using a scale of 1 cm to represent 1 unit on both axes**, draw the triangle ABC with vertices A (2,3), B (5,3) and C (3,6). **(3 marks)**

(ii) On the same axes used in (b) (i), draw and label the line $y = 2$. **(1 mark)**

(iii) Draw the image of triangle ABC under a reflection in the line $y = 2$. Label the image $A'B'C'$. **(2 marks)**

(iv) Draw a new triangle $A''B''C''$ with vertices A'' (–7,4), B'' (–4,4) and C'' (–6,7). **(1 mark)**

(v) Name and describe the single transformation that maps triangle ABC onto triangle $A''B''C''$. **(2 marks)**

Total 12 marks

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6. A class of 26 students each recorded the distance travelled to school. The distance, to the nearest km, is recorded below:

21	11	3	22	6	32	22	18	28
26	16	17	34	12	25	8	19	14
39	17	22	24	30	18	13	23	

(a) Copy and complete the frequency table to represent this data.

Distance in kilometres	**Frequency**
1 - 5	1
6 - 10	2
11 - 15	4
16 - 20	6
21 - 25	
26 - 30	
31 - 35	
36 - 40	

(2 marks)

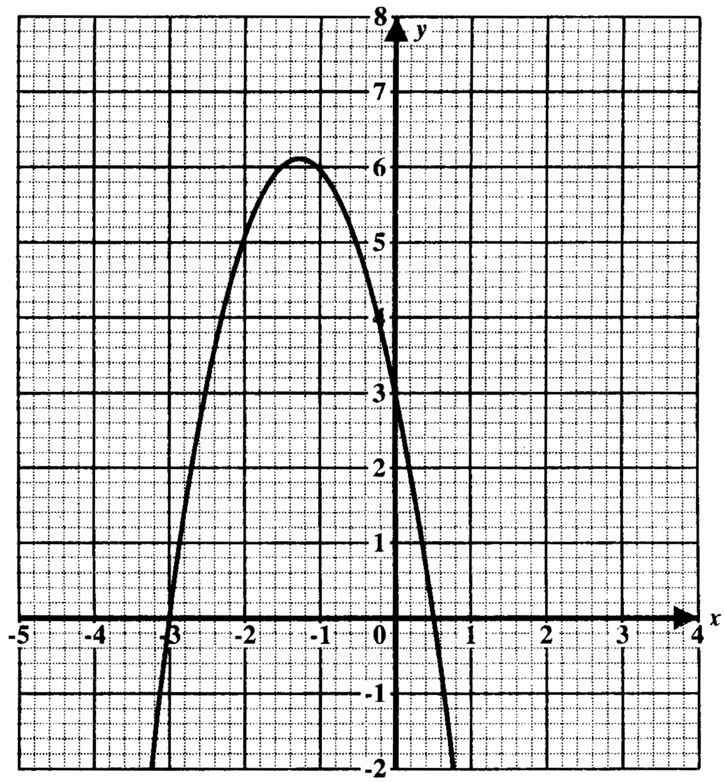
(b) **Using a scale of 2 cm to represent 5 km** on the horizontal axis and **a scale of 1 cm to represent 1 student** on the vertical axis, draw a histogram to represent the data.
(5 marks)

(c) Calculate the probability that a student chosen at random from this class recorded the distance travelled to school as 26 km or **more**. **(2 marks)**

(d) The P.T.A. plans to set up a transportation service for the school. Which average, mean, mode or median, is MOST appropriate for estimating the cost of the service? Give a reason for your answer. **(2 marks)**

Total 11 marks

7. The graph shown below represents a function of the form: $f(x) = ax^2 + bx + c$.



Using the graph above, determine

(i) the value of $f(x)$ when $x = 0$ **(1 mark)**

(ii) the values of x when $f(x) = 0$ **(2 marks)**

(iii) the coordinates of the maximum point **(2 marks)**

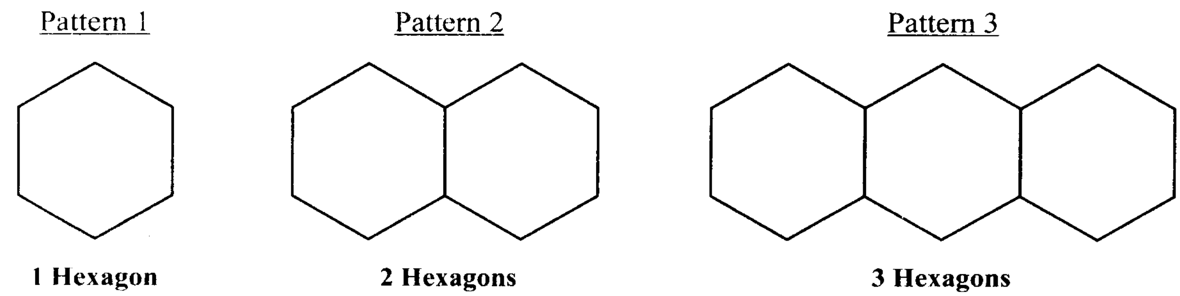
(iv) the equation of the axis of symmetry **(2 marks)**

(v) the values of x when $f(x) = 5$ **(2 marks)**

(vi) the interval within which x lies when $f(x) > 5$. Write your answer in the form $a < x < b$. **(2 marks)**

Total 11 marks

8. Bianca makes hexagons using sticks of equal length. She then creates patterns by joining the hexagons together. Patterns 1, 2 and 3 are shown below:



The table below shows the number of hexagons in EACH pattern created and the number of sticks used to make EACH pattern.

Number of hexagons in the pattern	1	2	3	4	5	20	n
Number of sticks used for the pattern	6	11	16	x	y	z	S

(a) Determine the values of

(i) x **(2 marks)**

(ii) y **(2 marks)**

(iii) z. **(2 marks)**

(b) Write down an expression for S in terms of n, where S represents the number of sticks used to make a pattern of n hexagons. **(2 marks)**

(c) Bianca used a total of 76 sticks to make a pattern of h hexagons. Determine the value of h. **(2 marks)**

Total 10 marks

SECTION II

Answer TWO questions in this section.

RELATIONS, FUNCTIONS AND GRAPHS

9. (a) The relationship between kinetic energy, E, mass, m, and velocity, v, for a moving particle is

$$E = \frac{1}{2}mv^2 .$$

(i) Express v in terms of E and m. **(3 marks)**

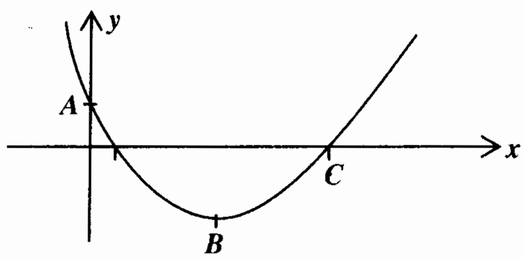
(ii) Determine the value of v when $E = 45$ and $m = 13$. **(2 marks)**

(b) Given $g(x) = 3x^2 - 8x + 2$,

(i) write $g(x)$ in the form $a(x + b)^2 + c$, where a, b and $c \in \mathbf{R}$ **(3 marks)**

(ii) solve the equation $g(x) = 0$, writing your answer(s) correct to 2 decimal places. **(4 marks)**

(iii) A sketch of the graph of $g(x)$ is shown below.



Copy the sketch and state

a) the y-coordinate of A

b) the x-coordinate of C

c) the x and y-coordinates of B. **(3 marks)**

Total 15 marks

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10. (a) The manager of a pizza shop wishes to make x small pizzas and y large pizzas. His oven holds no more than 20 pizzas.

(i) Write an inequality to represent the given condition. **(2 marks)**

The ingredients for each small pizza cost \$15 and for each large pizza \$30. The manager plans to spend no more than \$450 on ingredients.

(ii) Write an inequality to represent this condition. **(2 marks)**

(b) (i) **Using a scale of 2 cm on the x-axis to represent 5 small pizzas** and **2 cm on the y-axis to represent 5 large pizzas**, draw the graphs of the lines associated with the inequalities at (a) (i) and (a) (ii) above. **(4 marks)**

(ii) Shade the region which is defined by ALL of the following combined:

– the inequalities written at (a) (i) and (a) (ii)

– the inequalities $x \geq 0$ and $y \geq 0$ **(1 mark)**

(iii) Using your graph, state the coordinates of the vertices of the shaded region. **(2 marks)**

(c) The pizza shop makes a profit of \$8 on the sale of EACH small pizza and \$20 on the sale of EACH large pizza. All the pizzas that were made were sold.

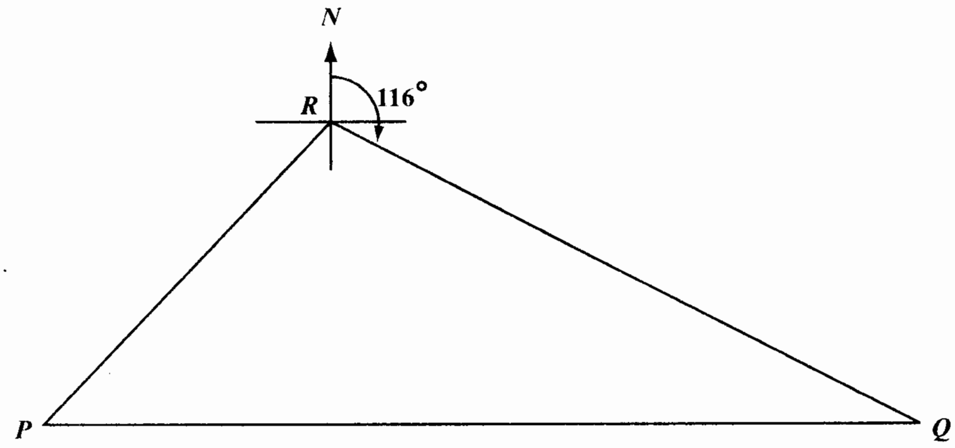
(i) Write an expression in x and y for the TOTAL profit made on the sale of the pizzas. **(1 mark)**

(ii) Use the coordinates of the vertices given at (b) (iii) to determine the MAXIMUM profit. **(3 marks)**

Total 15 marks

GEOMETRY AND TRIGONOMETRY

11. (a) The diagram below, **not drawn to scale**, shows three stations P, Q and R, such that the bearing of Q from R is 116° and the bearing of P from R is 242°. The vertical line at R shows the North direction.



(i) Show that angle $PRQ = 126°$. **(2 marks)**

(ii) Given that $PR = 38$ metres and $QR = 102$ metres, calculate the distance PQ, giving your answer to the nearest metre. **(3 marks)**

(b) K, L and M are points along a straight line on a horizontal plane, as shown below.

A vertical pole, SK, is positioned such that the angles of elevation of the top of the pole S from L and M are 21° and 14° respectively.

The height of the pole, SK, is 10 metres.

(i) Copy and complete the diagram to show the pole SK and the angles of elevation of S from L and M. **(4 marks)**

(ii) Calculate, correct to ONE decimal place,

a) the length of KL

b) the length of LM. **(6 marks)**

Total 15 marks

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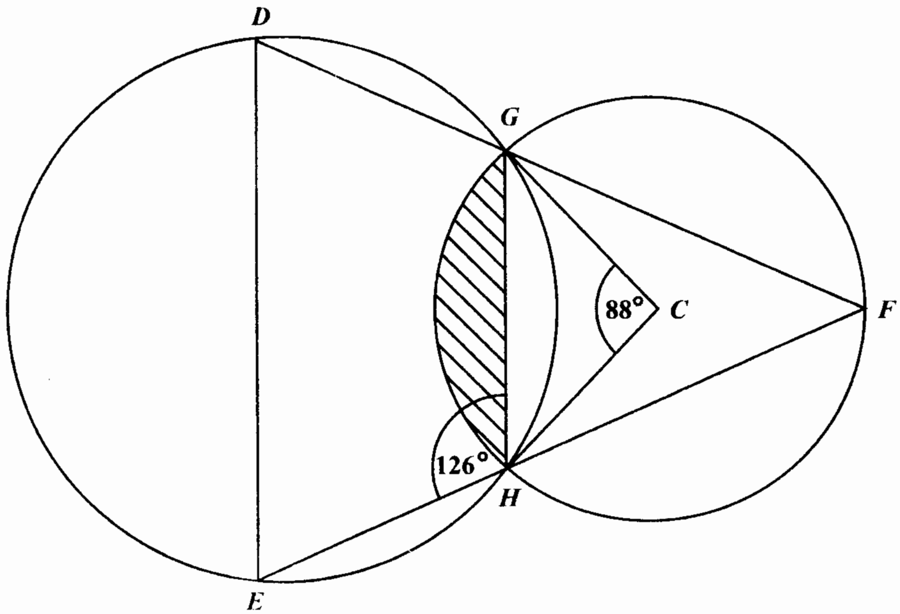
12. (a) The diagram below, **not drawn to scale**, shows two circles. *C* is the centre of the smaller circle, *GH* is a common chord and *DEF* is a triangle.

Angle $GCH = 88°$ and angle $GHE = 126°$.



Calculate, **giving reasons for your answer**, the measure of angle

(i) *GFH* **(2 marks)**

(ii) *GDE* **(3 marks)**

(iii) *DEF*. **(2 marks)**

(b) **Use $\pi = 3.14$ in this part of the question.**

Given that $GC = 4$ cm, calculate the area of

(i) triangle *GCH* **(3 marks)**

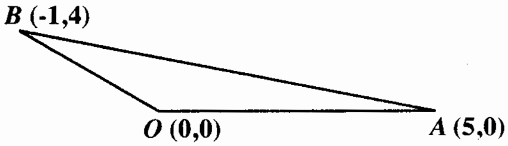
(ii) the minor sector bounded by arc *GH* and radii *GC* and *HC* **(3 marks)**

(iii) the shaded segment. **(2 marks)**

Total 15 marks

VECTORS AND MATRICES

13. (a) The figure below, **not drawn to scale**, shows the points O (0,0), A (5,0) and B (−1,4) which are the vertices of a triangle OAB.



(i) Express in the form $\begin{pmatrix} a \\ b \end{pmatrix}$ the vectors

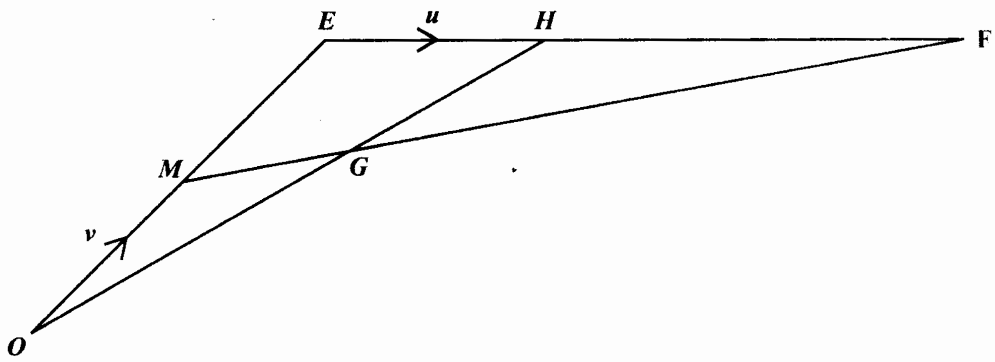
a) $\overrightarrow{OB}$

b) $\overrightarrow{OA} + \overrightarrow{OB}$ **(3 marks)**

(ii) If $M\,(x,y)$ is the midpoint of AB, determine the values of x and y. **(2 marks)**

(b) In the figure below, **not drawn to scale**, OE, EF and MF are straight lines. The point H is such that $EF = 3EH$. The point G is such that $MF = 5MG$. M is the midpoint of OE.

The vector $\overrightarrow{OM} = v$ and $\overrightarrow{EH} = u$.



(i) Write in terms of $\boldsymbol{u}$ and/or $\boldsymbol{v}$, an expression for:

a) $\overrightarrow{HF}$ **(1 mark)**

b) $\overrightarrow{MF}$ **(2 marks)**

c) $\overrightarrow{OH}$ **(2 marks)**

(ii) Show that $\overrightarrow{OG} = \frac{3}{5}\left(2v + u\right)$ **(2 marks)**

(iii) Hence, prove that O, G and H lie on a straight line. **(3 marks)**

Total 15 marks

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14. (a) L and N are two matrices where

$$L = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \quad \text{and} \quad N = \begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}.$$

Evaluate $L - N^2$. **(3 marks)**

(b) The matrix, M, is given as $M = \begin{pmatrix} x & 12 \\ 3 & x \end{pmatrix}$. Calculate the values of x for which M is singular. **(2 marks)**

(c) A geometric transformation, R, maps the point $(2,1)$ onto $(-1,2)$.

Given that $R = \begin{pmatrix} 0 & p \\ q & 0 \end{pmatrix}$, calculate the values of p and q. **(3 marks)**

(d) A translation, $T = \begin{pmatrix} r \\ s \end{pmatrix}$ maps the point $(5,3)$ onto $(1,1)$. Determine the values of r and s.

(3 marks)

(e) Determine the coordinates of the image of $(8,5)$ under the combined transformation, R followed by T. **(4 marks)**

Total 15 marks

END OF TEST