STRAND B: NUMBER THEORY

Unit 6 *Indices and Factors*

Student Text

Contents

Section

6 Indices and Factors

6.1 Squares, Cubes, Square Roots and Cube Roots

When a number is multiplied by itself, we say that the number has been *squared*.

For example, 3 squared means $3 \times 3 = 9$. This is written as $3^2 = 9$.
We could also say that 9 is the square of 3.

When a number is *cubed* it is written down 3 times and multiplied.

For example 2 cubed means $2 \times 2 \times 2 = 8$. This is written as $2^3 = 8$.
We could also say that 8 is the cube of 2.

Sometimes the reverse process is needed to answer questions such as:

What number squared gives 25?

The answer would be 5. We say that 5 is the square root of 25, or write $\sqrt{25} = 5$.
Another question might be:

What number cubed gives 8?

The answer would be 2. We would say that the cube root of 8 is 2.

We could also write $\sqrt[3]{8} = 2$.

Worked Example 1

Find

(a) 8^2 (b) 4^2 (c) 5^3.

Use your answers to find

(d) $\sqrt{64}$ (e) $\sqrt{16}$ (f) $\sqrt[3]{125}$

Solution

(a) $8^2 = 8 \times 8 = 64$

(b) $4^2 = 4 \times 4 = 16$

(c) $5^3 = 5 \times 5 \times 5 = 125$

(d) $\sqrt{64} = 8$ because $8^2 = 64$

(e) $\sqrt{16} = 4$ because $4^2 = 16$

(f) $\sqrt[3]{125} = 5$ because $5^3 = 125$

Exercises

1. Find

(a) 5^2 (b) 6^2 (c) 1^2 (d) 7^2

Use your answers to find

(e) $\sqrt{36}$ (f) $\sqrt{1}$ (g) $\sqrt{49}$ (h) $\sqrt{25}$

2. Find

(a) 3^3 (b) 4^3 (c) 6^3 (d) 10^3

Use your answers to find

(e) $\sqrt[3]{27}$ (f) $\sqrt[3]{1000}$ (g) $\sqrt[3]{216}$ (h) $\sqrt[3]{64}$

3. Find

(a) 10^2 (b) 2^2 (c) 4^2 (d) 7^2

(e) 8^2 (f) 9^2 (g) 1^3 (h) 7^3

(i) 8^3 (j) 0^2 (k) 0^3 (l) 2^3

4. Find

(a) $\sqrt{100}$ (b) $\sqrt{4}$ (c) $\sqrt{81}$ (d) $\sqrt{64}$

(e) $\sqrt{16}$ (f) $\sqrt{9}$

5. Use a calculator to find

(a) 12^2 (b) 11^2 (c) 15^3 (d) 13^3

(e) 13^2 (f) 15^2 (g) 20^2 (h) 11^3

Without a calculator, find

(i) $\sqrt{121}$ (j) $\sqrt{400}$ (k) $\sqrt{169}$ (l) $\sqrt{225}$

(m) $\sqrt[3]{3375}$ (n) $\sqrt[3]{2197}$ (o) $\sqrt{144}$ (p) $\sqrt[3]{1331}$

6. Find

(a) $6^2 + 4^2$ (b) $3^2 - 2^2$ (c) $10^2 + 4^2$ (d) $3^2 + 4^2$

(e) $5^2 - 3^2$ (f) $4^3 + 2^3$ (g) $1^3 + 10^3$ (h) $6^2 + 8^2$

Information

On average, a human heart beats 75 times a minute, 4 500 times an hour, 108 000 times a day, 39 420 000 times a year and 3 153 600 000 times for someone who lives 80 years.

6.2 Index Notation

Index notation is a very useful way of writing expressions like

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

in a shorter format. The above could be written with index notation as 2^7.
The small number, 7, is called the *index* or *power*.

Worked Example 1

Find (a) 3^4 (b) 4^5 (c) 7^1

Solution

(a) $3^4 = 3 \times 3 \times 3 \times 3$
$= 81$

(b) $4^5 = 4 \times 4 \times 4 \times 4 \times 4$
$= 1024$

(c) $7^1 = 7$

Worked Example 2

Find the missing number.

(a) $3^4 \times 3^6 = 3^?$ (b) $4^2 \times 4^3 = 4^?$ (c) $\dfrac{5^7}{5^4} = 5^?$

Solution

(a) $3^4 \times 3^6 = (3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3 \times 3 \times 3)$
$= 3^{10}$

(b) $4^2 \times 4^3 = (4 \times 4) \times (4 \times 4 \times 4)$
$= 4^5$

(c) $\dfrac{5^7}{5^4} = \dfrac{5 \times 5 \times 5 \times \overset{1}{\cancel{5}} \times \overset{1}{\cancel{5}} \times \overset{1}{\cancel{5}} \times \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \times \underset{1}{\cancel{5}} \times \underset{1}{\cancel{5}} \times \underset{1}{\cancel{5}}}$
$= 5 \times 5 \times 5$
$= 5^3$

Note

$$\boxed{a^m \times a^n = a^{m+n}} \quad \text{and} \quad \boxed{\frac{a^n}{a^m} = a^{n-m}}$$

These rules apply whenever index notation is used.

Using these rules,

$$\frac{a^3}{a^3} = a^{3-3} = a^0 \qquad \text{or} \qquad \frac{a^3}{a^3} = \frac{\overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}}}{\underset{1}{\cancel{a}} \times \underset{1}{\cancel{a}} \times \underset{1}{\cancel{a}}} = 1$$

So $\boxed{a^0 = 1}$

Worked Example 3

Find

(a) $\left(2^3\right)^4$ (b) $\left(3^2\right)^3$

Solution

(a) $\left(2^3\right)^4 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^{12}$

(b) $\left(3^2\right)^3 = (3 \times 3) \times (3 \times 3) \times (3 \times 3)$

$= 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 3^6$

Note

$$\boxed{\left(a^m\right)^n = a^{m \times n}}$$

Exercises

1. Write each of the following using index notation.

(a) $4 \times 4 \times 4 \times 4 \times 4$ (b) $3 \times 3 \times 3$

(c) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$ (d) $7 \times 7 \times 7 \times 7$

(e) $18 \times 18 \times 18$ (f) 19×19

(g) $4 \times 4 \times 4 \times 4 \times 4 \times 4$ (h) $7 \times 7 \times 7 \times 7 \times 7$

(i) $10 \times 10 \times 10 \times 10 \times 10 \times 10$ (j) $100 \times 100 \times 100 \times 100 \times 100$

2. Find the value of each of the following.

(a) 3^4 (b) 5^4 (c) 7^4 (d) 10^4

(e) 5^0 (f) 3^6 (g) 2^7 (h) 2^1

(i) 8^4 (j) 4^1 (k) 3^0 (l) 5^2

3. Fill in the missing numbers.

(a) $2^7 \times 2^4 = 2^?$ (b) $3^4 \times 3^5 = 3^?$ (c) $3^6 \times 3^7 = 3^?$

(d) $4^? \times 4^2 = 4^7$ (e) $5^? \times 5^2 = 5^6$ (f) $5^4 \times 5^? = 5^9$

(g) $?^2 \times 4^4 = 4^6$ (h) $5^7 \div 5^4 = 5^?$ (i) $3^4 \div 3^2 = 3^?$

(j) $7^{14} \div 7^{10} = 7^?$ (k) $17^5 \div 17^? = 17^3$ (l) $9^7 \div 9^? = 9^3$

(m) $4^6 \times 4^? = 4^{11}$ (n) $4^? \div 4^6 = 4^{10}$ (o) $3^? \times 3^2 = 3^8$

(p) $3^6 \div 3^6 = ?$ (q) $3^7 \div 3^6 = ?$ (r) $3^0 \times 3^? = 3^5$

(s) $3^0 \times 3^7 = 3^?$ (t) $4^1 \times 4^? = 4^8$ (u) $5^2 \times 5^? = 5^2$

4. Fill in the missing numbers.

(a) $4 = 2^?$ (b) $8 = 2^?$ (c) $16 = 2^?$

(d) $64 = 2^?$ (e) $27 = 3^?$ (f) $25 = 5^?$

(g) $64 = 4^?$ (h) $81 = 3^?$ (i) $125 = ?^3$

5. Simplify the following expressions, giving your answer in index notation.

(a) $3^7 \times 3^6 =$ (b) $2 \times 2^7 =$ (c) $4^5 \times 4^6 =$

(d) $3^6 \times 3^4 =$ (e) $2^4 \times 2^5 =$ (f) $2^6 \times 2^4 =$

(g) $3^7 \div 3^2 =$ (h) $3 \times 3^6 =$ (i) $3^6 \div 3 =$

(j) $\frac{8^{12}}{8^2} =$ (k) $\frac{7^6}{7^3} =$ (l) $\frac{9^2}{9^0} =$

(m) $4 \times 2^2 =$ (n) $\frac{2^5}{4} =$ (o) $\frac{2^6}{8} =$

6. Fill in the missing powers.

(a) $8 = 2^?$ (b) $1000 = 10^?$ (c) $16 = 2^?$

(d) $27 = 3^?$ (e) $81 = 3^?$ (f) $10\,000 = 10^?$

(g) $625 = 5^?$ (h) $64 = 4^?$ (i) $1296 = 6^?$

(j) $1 = 2^?$ (k) $36 = 6^?$ (l) $1 = 5^?$

7. Simplify the following, giving your answers in index form.

(a) $\left(2^3\right)^2 =$ (b) $\left(3^2\right)^2 =$ (c) $\left(6^2\right)^3 =$

(d) $\left(5^3\right)^2 =$ (e) $\left(2^2\right)^4 =$ (f) $\left(4^2\right)^3 =$

(g) $\left(3^2\right)^4 =$ (h) $\left(5^2\right)^4 =$ (i) $\left(3^3\right)^2 =$

8. Fill in the missing numbers.

(a) $\left(2^2\right)^4 = 2^?$ (b) $\left(2^?\right)^3 = 2^{12}$ (c) $\left(3^2\right)^5 = ?^{10}$

(d) $\left(5^?\right)^4 = 5^{12}$ (e) $\left(10^5\right)^? = 10^{15}$ (f) $\left(7^5\right)^? = 7^{20}$

9. Simplify each of the following, giving your answer in index notation.

(a) $3^2 \times 3^0 \times 3^4 =$ (b) $2^6 \times 2^7 \times 2 =$ (c) $5^2 \times 5^7 \times 5^3 =$

(d) $\dfrac{7^2 \times 7^4}{7^3} =$ (e) $\dfrac{7^4 \times 7^5}{7^2 \times 7^3} =$ (f) $\dfrac{2^3 \times 2^8}{2^3 \times 2} =$

(g) $\dfrac{3^2 \times 3^3}{3^5} =$ (h) $\dfrac{4^7 \times 4^8}{4^5 \times 4^9} =$ (i) $\dfrac{2^3 \times 2^0}{2^2} =$

10. Simplify each of the following expressions.

(a) $a^3 \times a^2 =$ (b) $a^4 \times a^6 =$ (c) $x^2 \times x^7 =$

(d) $x^4 \div x^2 =$ (e) $y^3 \times y^0 =$ (f) $p^7 \div p^4 =$

(g) $q^6 \div q^3 =$ (h) $x^7 \times x =$ (i) $b^4 \div b =$

(j) $\dfrac{b^6}{b^0} =$ (k) $\dfrac{c^7}{c^4} =$ (l) $\dfrac{x^8}{x^3} =$

(m) $\dfrac{y^3}{y} =$ (n) $\dfrac{x^4}{x^4} =$ (o) $x^2 \times x^3 \times x^3 =$

(p) $\dfrac{p^2 \times p^7}{p^5} =$ (q) $\dfrac{x^{10}}{x^2 \times x^5} =$ (r) $\dfrac{y^3 \times y^7}{y^2 \times y^4} =$

(s) $\dfrac{x^2 \times x^3}{x^5} =$ (t) $\dfrac{x^7 \times x}{x^3 \times x^4} =$ (u) $\dfrac{x^8 \times x^4}{x^0} =$

(v) $\left(x^2\right)^4 =$ (w) $\left(x^3\right)^5 =$ (x) $\left(x^2 \times x^7\right)^6 =$

11. 243 can be written as 3^5.

Find the values of p and q in the following:

(a) $64 = 4^p$ (b) $5^q = 1$

12. Express as simply as possible:

$$\frac{4x^2 \times 6x^5}{12x^3}$$

Challenge!

You open a book. Two pages face you. If the product of the two page numbers is 3 192, what are the two page numbers?

6.3 Factors

A factor of a positive whole number is a positive whole number that will divide exactly into it.

Worked Example 1

List all the factors of 20.

Solution

The factors of 20 are:

$$1,\ 2,\ 4,\ 5,\ 10,\ 20$$

These are all numbers that divide exactly into 20.

Worked Example 2

Write the number 12 as the product of two factors in as many ways as possible.

Solution

$$12 = 1 \times 12 \qquad 12 = 4 \times 3 \qquad 12 = 2 \times 6$$
$$12 = 6 \times 2 \qquad 12 = 3 \times 4 \qquad 12 = 12 \times 1$$

Exercises

1. List the factors of these numbers.

   (a) 14 (b) 27 (c) 6 (d) 15
   (e) 18 (f) 25 (g) 40 (h) 100
   (i) 45 (j) 50 (k) 36 (l) 28

2. Write each number below as the product of two factors in as many ways as possible.

   (a) 10 (b) 8 (c) 7 (d) 9
   (e) 16 (f) 22 (g) 11 (h) 24

3. Fill in the missing numbers.

   (a) $32 = 4 \times 2 \times ?$ (b) $45 = ? \times 3 \times 5$
   (c) $27 = 3 \times 3 \times ?$ (d) $40 = 5 \times ? \times 2$
   (e) $50 = 5 \times 2 \times ?$ (f) $88 = 11 \times 2 \times ?$
   (g) $66 = 2 \times 3 \times ?$ (h) $21 = ? \times 3 \times 7$

4. Here is a Bingo card.

6		10		20		9	
	3		8		17		15
2		24		55		4	

(a) Circle those numbers that 2 will divide into exactly.

(b) Cross out those numbers that 5 will divide into exactly.

5. 20 21 22 23 24 25 26 27 29

(a) In the row of numbers above:

(i) *circle* all numbers divisible by 2, e.g. (20)

(ii) *cross out* all numbers divisible by 3, e.g. ~~24~~

(iii) *underline* all numbers divisible by 5. e.g. <u>25</u>

(b) Describe the numbers which are not circled, crossed out or underlined.

6. A pattern of counting numbers is shown.

14, 15, 16, 17, 18, 19, 20, ...

(a) (i) Which of these numbers is a square number?

(ii) Which of these numbers is a multiple of nine?

The pattern is continued.

(b) (i) What is the next square number?

(ii) What is the next number that is a multiple of nine?

6.4 Prime Factors, HCF and LCM

Any positive whole number can be written as the product of a number of prime factors. For example,

$$20 = 2^2 \times 5$$

or

$$180 = 2^2 \times 3^2 \times 5$$

Note

A *prime number* is a positive whole number with exactly two factors; 1 and itself.

The first few prime numbers are 2, 3, 5, 7, 11, ...

Worked Example 1

Write the number 276 as a product of prime numbers.

Solution

Write 276 as a product of two factors:

$$276 = 2 \times 138$$

But $138 = 2 \times 69$ so $276 = 2 \times 2 \times 69$

But $69 = 3 \times 23$ so $276 = 2 \times 2 \times 3 \times 23$

This expression contains only prime numbers, so

$$276 = 2^2 \times 3 \times 23$$

This is called the *product of prime factors*.

Another important concept is that of the *highest common factor* (HCF) of two (or more) positive integers. The HCF is the largest number which is a factor of both (or all) the numbers.

Worked Example 2

Find the HCF of 120 and 105.

Solution

Expressing both 120 and 105 in terms of their prime factors gives

$$120 = 2 \times 2 \times 2 \times 3 \times 5$$

$$105 = 3 \times 5 \times 7$$

It is easy now to see that the highest common factor is $3 \times 5 = 15$.

Worked Example 3

(a) Write the numbers 660 and 470 as the product of prime factors.

(b) Find the largest common factor that will divide into both 660 and 470.

Solution

(a)

$$\begin{aligned} 660 &= 2 \times 330 \\ &= 2 \times 2 \times 165 \\ &= 2 \times 2 \times 3 \times 55 \\ &= 2 \times 2 \times 3 \times 5 \times 11 \end{aligned}$$

So as a product of prime factors,

$$660 = 2^2 \times 3 \times 5 \times 11$$

$$\begin{aligned} 470 &= 2 \times 235 \\ &= 2 \times 5 \times 47 \end{aligned}$$

So as a product of prime factors,

$$470 = 2 \times 5 \times 47$$

(b) To find the largest common factor that will divide into both 660 and 470, look at the factors common to each of the products of primes.

The numbers that appear in both are 2 and 5, so the largest number that will divide into both 660 and 470 is $2 \times 5 = 10$.

So 10 is the HCF of 660 and 470.

A related concept is that of the *lowest common multiple*, LCM, of two (or more) positive integers. This is the lowest number into which the two (or all) numbers can divide exactly.

Worked Example 4

Find the LCM of 24 and 60.

Solution

One way to find the LCM is to write out multiples of each number.

For example,

multiples of 24 are 24, 48, 72, 96, (120), 144, 168, 192, 216, 240, ...

multiples of 60 are 60, (120), 180, 240, 200, 360, ...

It is easy to see that 120 is the LCM of 24 and 60.

Another way is to express 24 and 60 in terms of their prime factors:

$$24 = 2 \times 2 \times 2 \times 3 = 2 \times (2 \times 2 \times 3)$$

$$60 = 2 \times 2 \times 3 \times 5 = 5 \times (2 \times 2 \times 3)$$

Noting that $(2 \times 2 \times 3)$ is common to both numbers, the LCM is given by

$5 \times 24 = 120$ or $2 \times 60 = 120$. So the LCM $= 120$

Note that $120 \div 24 = 5$ and $120 \div 60 = 2$.

Exercises

1. Which of the following are prime numbers?

   1, 2, 3, 5, 7, 9, 13, 15, 18, 19, 21, 23, 25

2. Which numbers between 50 and 60 are prime numbers?

3. Write each number below as a product of prime factors.

   (a) 10 (b) 42 (c) 68
   (d) 168 (e) 250 (f) 270
   (g) 429 (h) 825 (i) 1001

4. (a) Express 32 and 56 as the product of prime factors.

   (b) By comparing the answers to (a) find the HCF of 32 and 56.

5. Find the highest common factors of each pair of numbers below.

   (a) 36, 42 (b) 30, 42 (c) 45, 105
   (d) 42, 50 (e) 50, 80 (f) 70, 315
   (g) 216, 240 (h) 156, 234 (i) 735, 1617

6. (a) Express each of the following numbers as the product of prime factors:

$$45,\ 99,\ 135$$

(b) By considering the products of the prime factors, find the highest common factor of

(i) 45 and 99 (ii) 99 and 135 (iii) 45 and 135

(c) What is the highest common factor of all three numbers?

7. Find the HCF for each set of three numbers given below.

(a)	20, 35, 105	(b)	90, 225, 405	(c)	16, 24, 56
(d)	200, 210, 220	(e)	72, 168, 312	(f)	330, 450, 630
(g)	216, 324, 432	(h)	660, 572, 528	(i)	1008, 1260, 1764

8. Find the LCM of

(a)	15 and 35	(b)	12 and 20	(c)	28 and 49
(d)	19 and 15	(e)	20 and 42	(f)	81 and 192

9. Find the LCM of each of the following sets of numbers.

(a)	8, 12, 40	(b)	36, 8, 12	(c)	25, 10,15
(d)	9, 8, 72	(e)	90, 80, 72	(f)	22, 10, 8

6.5 Further Index Notation

Indices can also be negative or fractions. The rules below explain how to use these types of indices.

$$a^{-1} = \frac{1}{a}$$ This is called the *reciprocal* of a.

$$a^{-n} = \frac{1}{a^n}$$

$$a^{\frac{1}{2}} = \sqrt{a}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Worked Example 1

Find:

(a)	2^{-4}	(b)	3^{-2}	(c)	5^{-1}
(d)	$4^{\frac{1}{2}}$	(e)	$8^{\frac{1}{3}}$	(d)	$9^{\frac{3}{2}}$

Solution

(a) $2^{-4} = \frac{1}{2^4}$

$= \frac{1}{2 \times 2 \times 2 \times 2}$

$= \frac{1}{16}$

(b) $3^{-2} = \frac{1}{3^2}$

$= \frac{1}{3 \times 3}$

$= \frac{1}{9}$

(c) $5^{-1} = \frac{1}{5}$

(d) $4^{\frac{1}{2}} = \sqrt{4}$

$= 2$

(e) $8^{\frac{1}{3}} = \sqrt[3]{8}$

$= 2$

(f) $9^{\frac{3}{2}} = \left(9^{\frac{1}{2}}\right)^3$

$= 3^3$

$= 3 \times 3 \times 3$

$= 27$

Worked Example 2

Find

(a) $2^{-5} \times 2^6$ (b) $m^2 \times m^{-4}$ (c) $\frac{3^{-7}}{3^2}$

(d) $\left(2^8 \times 2^6\right)^{\frac{1}{2}}$ (e) $\left(a^2 \times b^{-2}\right)^{-1}$ (f) $\left(\frac{m^2}{a}\right)^{-2}$

Solution

(a) $2^{-5} \times 2^6 = 2^{-5+6}$

$= 2^1$

$= 2$

(b) $m^2 \times m^{-4} = m^{2-4}$

$= m^{-2}$

$= \frac{1}{m^2}$

(c) $\frac{3^{-7}}{3^2} = 3^{-7-2}$

$= 3^{-9}$

$= \frac{1}{3^9}$

(d) $\left(2^8 \times 2^6\right)^{\frac{1}{2}} = \left(2^{8+6}\right)^{\frac{1}{2}}$

$= \left(2^{14}\right)^{\frac{1}{2}}$

$= 2^{14 \times \frac{1}{2}}$

$= 2^7$

(e) $\left(a^2 \times b^{-2}\right)^{-1} = a^{-2} \times b^2$

$= \dfrac{b^2}{a^2}$

(f) $\left(\dfrac{m^2}{a}\right)^{-2} = \left(m^2 a^{-1}\right)^{-2}$

$= m^{-4} a^2$

$= \dfrac{a^2}{m^4}$

Exercises

1. Find as fractions that do not involve indices, *without* using a calculator:

(a) $4^{-2} =$ (b) $2^{-3} =$ (c) $6^{-1} =$

(d) $7^{-1} =$ (e) $9^{\frac{1}{2}} =$ (f) $64^{\frac{1}{2}} =$

(g) $16^{\frac{1}{4}} =$ (h) $27^{\frac{1}{3}} =$ (i) $1^{\frac{1}{3}} =$

(j) $5^{-2} =$ (k) $16^{\frac{3}{4}} =$ (l) $4^{\frac{5}{2}} =$

(m) $9^{\frac{7}{2}} =$ (n) $25^{\frac{3}{2}} =$ (o) $8^{-\frac{1}{3}} =$

2. Complete the missing numbers, *without* using a calculator.

(a) $3^? = \dfrac{1}{81}$ (b) $2^? = \dfrac{1}{2}$ (c) $5^? = \dfrac{1}{125}$

(d) $36^? = 6$ (e) $36^? = \dfrac{1}{6}$ (f) $7^? = 49$

(g) $7^? = 343$ (h) $17^? = \dfrac{1}{17}$ (i) $125^? = 5$

(j) $\dfrac{1}{2} = 2^?$ (k) $\dfrac{1}{4} = 2^?$ (l) $\dfrac{1}{100} = 10^?$

(m) $\dfrac{1}{a^3} = a^?$ (n) $\sqrt{m} = m^?$ (o) $\dfrac{1}{p^2} = p^?$

(p) $\sqrt[3]{q} = q^?$ (q) $\sqrt[3]{q^2} = q^?$ (r) $\sqrt[5]{q^2} = q^?$

3. Use a calculator to find:

(a) 8^{-1} (b) 20^{-1} (c) $\left(\dfrac{1}{2}\right)^{-1}$ (d) $\left(\dfrac{1}{4}\right)^{-1}$

(e) 15^{-2} (f) 20^{-3} (g) $81^{\frac{3}{2}}$ (h) $243^{\frac{3}{5}}$

(i) $16^{-\frac{1}{4}}$ (j) $144^{\frac{3}{2}}$ (k) $169^{\frac{7}{2}}$ (l) $121^{\frac{3}{2}}$

4. Simplify the following expressions, so that they contain no negative indices.

(a) $a^6 \times a^{-7} =$ (b) $\dfrac{a^7}{a^{-3}} =$ (c) $\dfrac{a^{-5}}{a^{-9}} =$

(d) $a^{-4} \times a^{-2} =$ (e) $\left(a^2\right)^{-1}$ (f) $\left(a^2\right)^{-3} =$

(g) $\left(a^{-2}\right)^{-4} =$ (h) $\left(a^{\frac{1}{2}}\right)^{5} =$ (i) $\left(a^{3}\right)^{-\frac{1}{2}} =$

(j) $\left(a^{6}\right)^{\frac{1}{3}} =$ (k) $\left(a^{9}\right)^{-\frac{1}{3}} =$ (l) $\left(a^{-12}\right)^{-\frac{1}{4}} =$

(m) $\left(\dfrac{a}{b}\right)^{2} =$ (n) $\left(a^{2} \times b^{-4}\right)^{3} =$ (o) $\left(a^{3} b^{\frac{1}{2}}\right)^{4} =$

(p) $\left(a^{2} b^{-2}\right)^{-2} =$ (q) $\left(\dfrac{a^{2}}{b^{3}}\right)^{4} =$ (r) $\left(m^{-1} n^{3}\right)^{-2} =$

(s) $\left(\dfrac{a^{6}}{b^{10}}\right)^{\frac{1}{2}} =$ (t) $\left(\dfrac{a^{2}}{m^{4}}\right)^{-\frac{1}{2}} =$ (u) $\left(\dfrac{a^{8} b^{2}}{c^{6}}\right)^{-\frac{1}{2}} =$

(v) $\left(\dfrac{m^{2}}{x}\right)^{-1} =$ (w) $\left(\dfrac{x^{2} y}{z^{3}}\right)^{-4} =$ (x) $\left[\left(a^{3} b^{-8}\right)^{-\frac{1}{3}}\right]^{2} =$

5. (a) Express $81^{-\frac{1}{2}}$ as a fraction in the form $\dfrac{a}{b}$, where a and b are integers.

   (b) Simplify $a^{6} \div a^{2}$.

   (c) Find the value of y for which $2 \times 4^{y} = 64$.

Investigation

Find four integers, a, b, c and d such that $a^{3} + b^{3} + c^{3} = d^{3}$.