

TEST CODE **01234020**

**FORM TP 2007017**

JANUARY 2007

**CARIBBEAN EXAMINATIONS COUNCIL  
SECONDARY EDUCATION CERTIFICATE  
EXAMINATION  
MATHEMATICS**

**Paper 02 – General Proficiency**

*2 hours 40 minutes*

**03 JANUARY 2007 (a.m.)**

**INSTRUCTIONS TO CANDIDATES**

1. Answer ALL questions in Section I, and ANY TWO in Section II.
2. Write your answers in the booklet provided.
3. All working must be shown clearly.
4. A list of formulae is provided on page 2 of this booklet.

**Examination Materials**

Electronic calculator (non-programmable)

Geometry set

Mathematical tables (provided)

Graph paper (provided)

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**LIST OF FORMULAE**

Volume of a prism  $V = Ah$  where  $A$  is the area of a cross-section and  $h$  is the perpendicular length.

Volume of cylinder  $V = \pi r^2 h$  where  $r$  is the radius of the base and  $h$  is the perpendicular height.

Volume of a right pyramid  $V = \frac{1}{3} Ah$  where  $A$  is the area of the base and  $h$  is the perpendicular height.

Circumference  $C = 2\pi r$  where  $r$  is the radius of the circle.

Area of a circle  $A = \pi r^2$  where  $r$  is the radius of the circle.

Area of trapezium  $A = \frac{1}{2} (a + b) h$  where  $a$  and  $b$  are the lengths of the parallel sides and  $h$  is the perpendicular distance between the parallel sides.

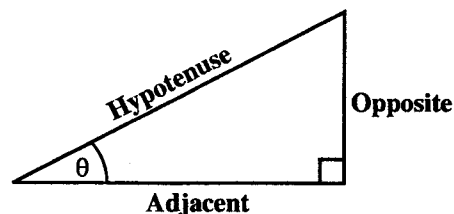
Roots of quadratic equations If  $ax^2 + bx + c = 0$ ,  
then  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



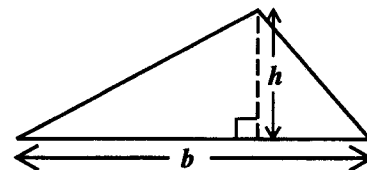
Area of triangle

Area of  $\Delta = \frac{1}{2} bh$  where  $b$  is the length of the base and  $h$  is the perpendicular height

$$\text{Area of } \Delta ABC = \frac{1}{2} ab \sin C$$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$

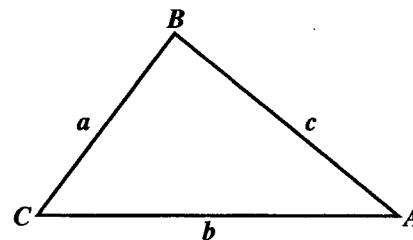


Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$



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## SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, evaluate
- (i)  $5.24(4 - 1.67)$  ( 2 marks)
- (ii)  $\frac{1.68}{1.5^2 - 1.45}$  ( 3 marks)
- (b) A sum of money is shared between Aaron and Betty in the ratio 2 : 5. Aaron received \$60. How much money was shared **altogether**? ( 3 marks)
- (c) In St. Vincent, 3 litres of gasoline cost EC\$10.40.
- (i) Calculate the cost of 5 litres of gasoline in St. Vincent, **stating your answer correct to the nearest cent.** ( 2 marks)
- (ii) How many litres of gasoline can be bought for EC \$50.00 in St. Vincent?  
Give your answer correct to the nearest whole number. ( 2 marks)

**Total 12 marks**

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2. (a) If  $a = 2$ ,  $b = -3$  and  $c = 4$ , evaluate

(i)  $ab - bc$  ( 1 mark )

(ii)  $b(a - c)^2$  ( 2 marks )

(b) Solve for  $x$  where  $x \in \mathbf{Z}$ :

(i)  $\frac{x}{2} + \frac{x}{3} = 5$  ( 3 marks )

(ii)  $4 - x \leq 13$  ( 3 marks )

(c) The cost of ONE muffin is  $\$m$ .  
The cost of THREE cupcakes is  $\$2m$ .

(i) Write an algebraic expression in  $m$  for the cost of:

a) FIVE muffins ( 1 mark )

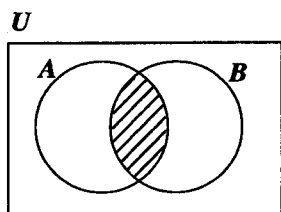
b) SIX cupcakes ( 1 mark )

(ii) Write an equation, in terms of  $m$ , to represent the following information.

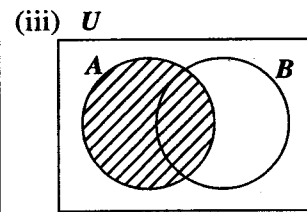
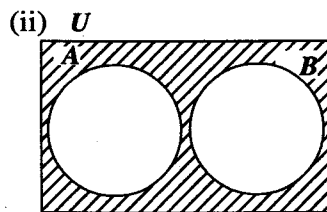
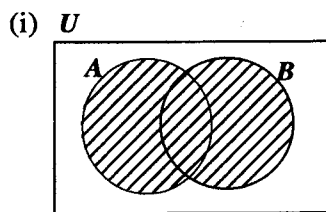
The TOTAL cost of 5 muffins and 6 cupcakes is  $\$31.50$ . ( 1 mark )

**Total 12 marks**

3. (a) Describe, using set notation only, the shaded regions in each Venn diagram below. The first one is done for you.



$A \cap B$



( 3 marks )

(b) The following information is given.

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$P = \{\text{prime numbers}\}$$

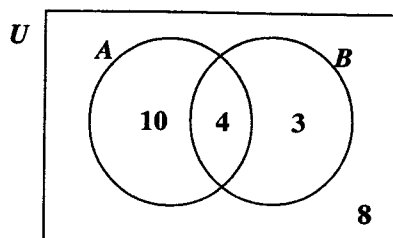
$$Q = \{\text{odd numbers}\}$$

Draw a Venn diagram to represent the information above.

( 3 marks )

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- (c) The Venn diagram below shows the number of elements in each region.



Determine how many elements are in EACH of the following sets:

- (i)  $A \cup B$  ( 1 mark )
- (ii)  $A \cap B$  ( 1 mark )
- (iii)  $(A \cap B)'$  ( 1 mark )
- (iv)  $U$  ( 1 mark )

**Total 10 marks**

4. (a) (i) Using a pencil, ruler and a pair of compasses only, construct  $\Delta ABC$  with  $BC = 6$  cm and  $AB = AC = 8$  cm. ( 3 marks )

**All construction lines must be clearly shown.**

- (ii) Draw a line segment  $AD$  such that  $AD$  meets  $BC$  at  $D$  and is perpendicular to  $BC$ . ( 2 marks )
- (iii) Measure and state
- a) the length of the line segment  $AD$  ( 1 mark )
- b) the size of angle  $ABC$  ( 1 mark )
- (b)  $P$  is the point  $(2, 4)$  and  $Q$  is the point  $(6, 10)$ .
- Calculate
- (i) the gradient of  $PQ$  ( 2 marks )
- (ii) the midpoint of  $PQ$ . ( 2 marks )

**Total 11 marks**

**5. An answer sheet is provided for this question.**

- (a)  $f$  and  $g$  are functions defined as follows

$$f: x \rightarrow 7x + 4$$

$$g: x \rightarrow \frac{1}{2x}$$

Calculate

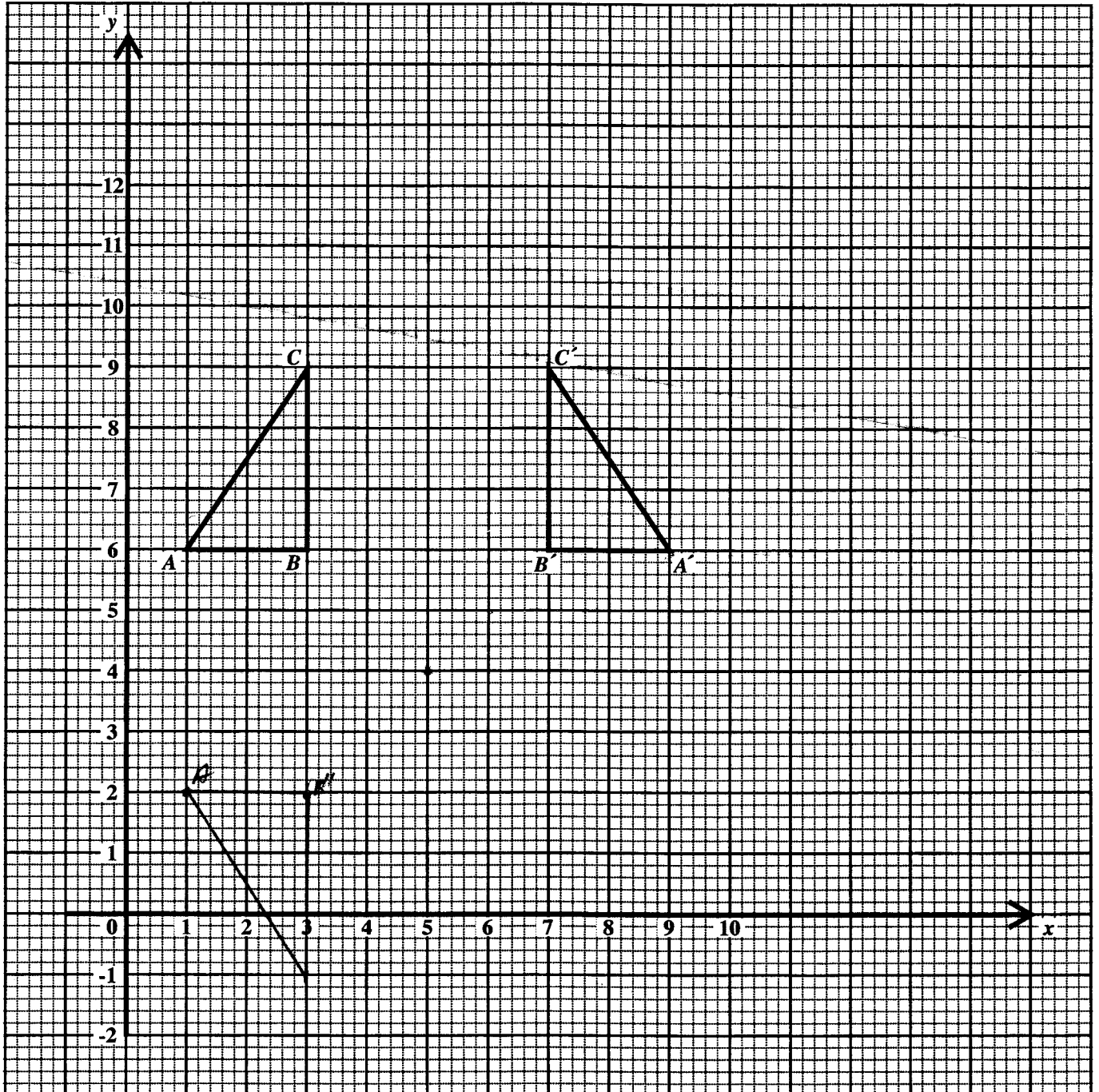
- (i)  $g(3)$  ( 1 mark )
- (ii)  $f(-2)$  ( 2 marks )
- (iii)  $f^{-1}(11)$  ( 2 marks )
- (b) On the answer sheet provided,  $\Delta ABC$  is mapped onto  $\Delta A'B'C'$  under a reflection.

- (i) Write down the equation of the mirror line. ( 1 mark )

$\Delta A'B'C'$  is mapped onto  $\Delta A''B''C''$  by a rotation of  $180^\circ$  about the point  $(5, 4)$ .

- (ii) Determine the coordinates of the vertices of  $\Delta A''B''C''$ . ( 3 marks )
- (iii) State the transformation that maps  $\Delta ABC$  onto  $\Delta A''B''C''$ . ( 2 marks )

**Total 11 marks**



ATTACH THIS ANSWER SHEET TO YOUR ANSWER BOOKLET

6. The table below shows a frequency distribution of the scores of 100 students in an examination.

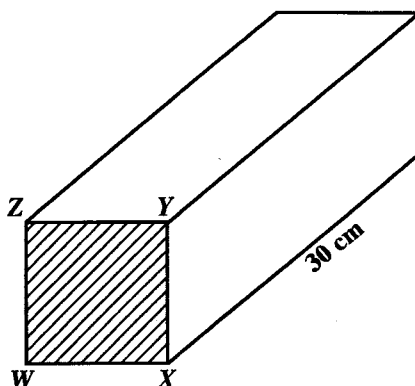
| Scores  | Frequency | Cumulative Frequency |
|---------|-----------|----------------------|
| 21 - 25 | 5         | 5                    |
| 26 - 30 | 18        |                      |
| 31 - 35 | 23        |                      |
| 36 - 40 | 22        |                      |
| 41 - 45 | 21        |                      |
| 46 - 50 | 11        | 100                  |

- (i) Copy and complete the table above to show the cumulative frequency for the distribution. ( 2 marks)
- (ii) Using a scale of 2 cm to represent a score of 5 on the horizontal axis and a scale of 2 cm to represent 10 students on the vertical axis, draw a cumulative frequency curve of the scores. Start your horizontal scale at 20. ( 6 marks)
- (iii) Using the cumulative frequency curve, determine the median score for the distribution. ( 2 marks)
- (iv) What is the probability that a student chosen at random has a score greater than 40? ( 2 marks)

**Total 12 marks**

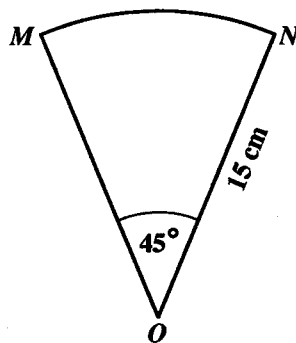


7. (a) The diagram below, **not drawn to scale**, shows a prism of length 30 cm. The cross-section  $WXYZ$  is a square with area  $144 \text{ cm}^2$ .



Calculate

- (i) the volume, in  $\text{cm}^3$ , of the prism ( 2 marks)
- (ii) the total surface area, in  $\text{cm}^2$ , of the prism. ( 4 marks)
- (b) The diagram below, **not drawn to scale**, shows the sector of a circle with centre  $O$ .  $\angle MON = 45^\circ$  and  $ON = 15 \text{ cm}$ .



Use  $\pi = 3.14$

Calculate, giving your answer correct to 2 decimal places

- (i) the length of the minor arc  $MN$  ( 2 marks)
- (ii) the perimeter of the figure  $MON$  ( 2 marks)
- (iii) the area of the figure  $MON$ . ( 2 marks)

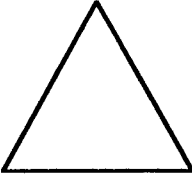
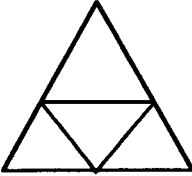
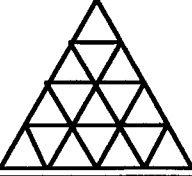
**Total 12 marks**

8. A large equilateral triangle is subdivided into a set of smaller equilateral triangles by the following procedure:

The midpoints of the sides of each equilateral triangle are joined to form a new set of smaller triangles.

The procedure is repeated  $n$  times.

The table below shows the results when the above procedure has been repeated twice, that is, when  $n = 2$ .

| $n$   | Result after each step   | No. of triangles formed |
|-------|--|-------------------------|
| 0     |   | 1                       |
| 1     |   | 4                       |
| 2     |  | 16                      |
| 3     |  | (i)                     |
| 6     |  | (ii)                    |
| (iii) |  | 65536                   |
| $m$   |  | (iv)                    |

(i) Calculate the number of triangles formed when  $n = 3$ . ( 2 marks)

(ii) Determine the number of triangles formed when  $n = 6$ . ( 2 marks)

A shape has 65 536 small triangles.

(iii) Calculate the value of  $n$ . ( 3 marks)

(iv) Determine the number of small triangles in a shape after carrying out the procedure  $m$  times. ( 3 marks)

**Total 10 marks**

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## SECTION II

Answer TWO questions in this section

## ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) Factorise completely
- (i)  $2p^2 - 7p + 3$  ( 1 mark )
- (ii)  $5p + 5q + p^2 - q^2$  ( 2 marks)
- (b) Expand  $(x + 3)^2(x - 4)$ , writing your answer in descending powers of  $x$ . ( 3 marks)
- (c) Given  $f(x) = 2x^2 + 4x - 5$
- (i) write  $f(x)$  in the form  $f(x) = a(x + b)^2 + c$  where  $a, b, c \in \mathbf{R}$  ( 3 marks)
- (ii) state the equation of the axis of symmetry ( 1 mark )
- (iii) state the coordinates of the minimum point ( 1 mark )
- (iv) sketch the graph of  $f(x)$  ( 2 marks)
- (v) on the graph of  $f(x)$  show clearly
- a) the minimum point ( 1 mark )
- b) the axis of symmetry. ( 1 mark )

**Total 15 marks**

**10. An answer sheet is provided for this question.**

Pam visits the stationery store where she intends to buy  $x$  pens and  $y$  pencils.

(a) Pam must buy at least 3 pens.

(i) Write an inequality to represent this information. ( 1 mark )

The TOTAL number of pens and pencils must NOT be more than 10.

(ii) Write an inequality to represent this information. ( 2 marks )

EACH pen costs \$5.00 and EACH pencil costs \$2.00. More information about the pens and pencils is represented by:

$$5x + 2y \leq 35$$

(iii) Write the information represented by this inequality as a sentence in your own words. ( 2 marks )

(b) (i) On the answer sheet provided, draw the graph of the TWO inequalities obtained in (a) (i) and (a) (ii) above. ( 3 marks )

(ii) Write the coordinates of the vertices of the region that satisfies the four inequalities (including  $y \geq 0$ ). ( 2 marks )

(c) Pam sells the  $x$  pens and  $y$  pencils and makes a profit of \$1.50 on EACH pen and \$1.00 on EACH pencil.

(i) Write an expression in  $x$  and  $y$  to represent the profit Pam makes. ( 1 mark )

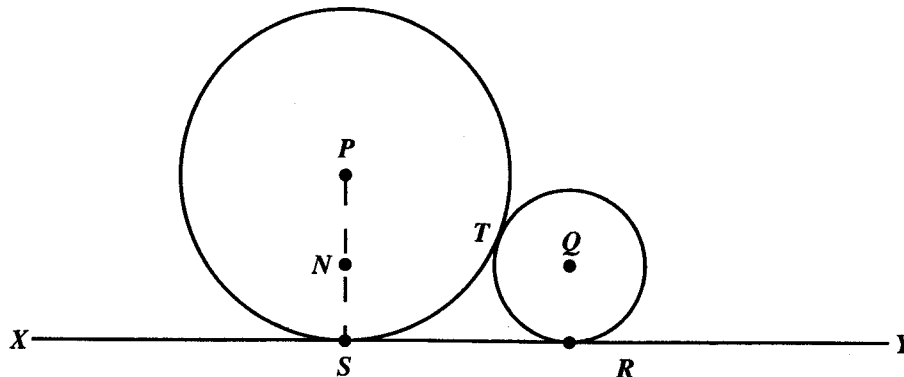
(ii) Calculate the maximum profit Pam makes. ( 2 marks )

(iii) If Pam buys 4 pens, show **on your graph** the maximum number of pencils she can buy. ( 2 marks )

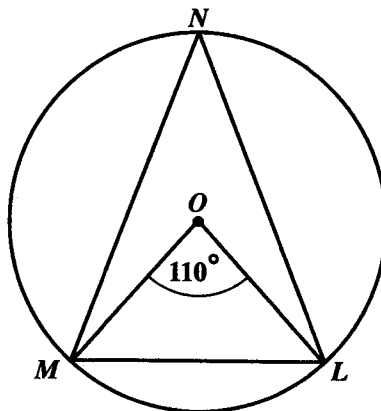
**Total 15 marks**

### GEOMETRY AND TRIGONOMETRY

11. (a) Two circles with centres  $P$  and  $Q$  and radii 5 cm and 2 cm respectively are drawn so that they touch each other at  $T$  and a straight line  $XY$  at  $S$  and  $R$ .



- (i) State, with a reason,
- why  $PTQ$  is a straight line ( 2 marks)
  - the length  $PQ$  ( 2 marks)
  - why  $PS$  is parallel to  $QR$ . ( 2 marks)
- (ii)  $N$  is a point on  $PS$  such that  $QN$  is perpendicular to  $PS$ .  
Calculate
- the length  $PN$  ( 2 marks)
  - the length  $RS$ . ( 2 marks)
- (b) In the diagram below, **not drawn to scale**,  $O$  is the centre of the circle. The measure of angle  $LOM$  is  $110^\circ$ .



Calculate, giving reasons for your answers, the size of EACH of the following angles

- $\angle MNL$  ( 2 marks)
- $\angle LMO$  ( 3 marks)

**Total 15 marks**

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12. A boat leaves a dock at point A and travels for a distance of 15 km to point B on a bearing of  $135^\circ$ .

The boat then changes course and travels for a distance of 8 km to point C on a bearing of  $060^\circ$ .

- (a) Illustrate the above information in a clearly labelled diagram. ( 2 marks)

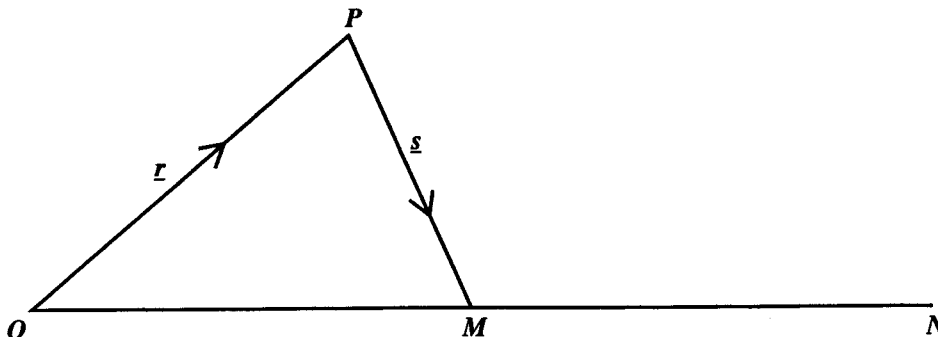
The diagram should show the

- (i) north direction ( 1 mark )
  - (ii) bearings  $135^\circ$  and  $060^\circ$  ( 2 marks)
  - (iii) distances 8 km and 15 km. ( 2 marks)
- (b) Calculate
- (i) the distance AC ( 3 marks)
  - (ii)  $\angle BCA$  ( 3 marks)
  - (iii) the bearing of A from C. ( 2 marks)

**Total 15 marks**

## VECTORS AND MATRICES

13. In the diagram below,  $M$  is the midpoint of  $\overrightarrow{ON}$ .



- (a) (i) Sketch the diagram above in your answer booklet and insert the point  $X$  on  $\overrightarrow{OM}$  such that  $\overrightarrow{OX} = \frac{1}{3} \overrightarrow{OM}$ . ( 1 mark )
- (ii) Produce  $PX$  to  $Q$  such that  $\overrightarrow{PX} = 4 \overrightarrow{XQ}$ . ( 1 mark )
- (b) Write the following in terms of  $\underline{r}$  and  $\underline{g}$ .
- (i)  $\overrightarrow{OM}$  ( 2 marks )
- (ii)  $\overrightarrow{PX}$  ( 3 marks )
- (iii)  $\overrightarrow{QM}$  ( 4 marks )
- (c) Show that  $\overrightarrow{PN} = 2 \overrightarrow{PM} + \overrightarrow{OP}$  ( 4 marks )

**Total 15 marks**

14. (a) Given that  $D = \begin{pmatrix} 1 & 9p \\ p & 4 \end{pmatrix}$  is a singular matrix, determine the value(s) of  $p$ .  
( 4 marks)

- (b) Given the linear equations

$$2x + 5y = 6$$

$$3x + 4y = 8$$

- (i) Write the equations in the form  $AX = B$  where  $A$ ,  $X$  and  $B$  are matrices.  
( 2 marks)
- (ii) a) Calculate the determinant of the matrix  $A$ .  
( 2 marks)

- b) Show that  $A^{-1} = \begin{pmatrix} \frac{-4}{7} & \frac{5}{7} \\ \frac{3}{7} & \frac{-2}{7} \end{pmatrix}$ .  
( 2 marks)

- c) Use the matrix  $A^{-1}$  to solve for  $x$  and  $y$ .  
( 5 marks)

**Total 15 marks**

**END OF TEST**