MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

04 JANUARY 2011 (a.m.)

1. Number
   a. 
      I. 86.65 
      II. V 
   b. Salaries 
      I. Basic wage for one $380 
      II. Overtime wage for six hours overtime $85.50 
      III. Total paid in overtime salaries $684 
      IV. Total overtime hours worked 48 

2. Algebra
   a. \( \frac{x}{15} \)
   b. \( ab(a + 2) \)
   c. \( p = \sqrt{ql + r} \)
   d. Donut problem 
      I. Expression \( 8x + 5(2x + 3) \)
      II. Equation, solving it gives 
      a. Small box \( x = 10 \)
      b. Large box \( 2x + 3 = 23 \)

3. 
   a. \( 12p^7q^4 \)
   b. Measurement 
      I. \( 240 \text{cm}^3 \)
      II. 13 cartons, since you can’t buy 12.5 
      \( V = \pi r^2h \)
      III. \[ h = \frac{240}{\pi \times 2.5^2} = 12.2 \text{cm} \]
4. 
   a. Sets
      I. $H = \{5, 7, 9, 11\}$
      II. $J = \{2, 3, 5, 7, 11\}$
      III. Draw the diagram to show the information

\[ U \]

\[ \begin{array}{c}
H \\
| \\
| \\
| \\
| \\
| 9 \\
| \\
| 7 \\
| \\
| 11 \\
| \\
| 2 \\
| \\
| 3 \\
| \\
| 5 \\
| \\
| 1, 4, 6, 8, 10, 12 \\
\end{array} \]

b. Part b is deliberately left out

5. Functions/coordinate geometry
   
   $3y = 2x - 6 \Rightarrow y = mx + c$

   a. 
      $y = \frac{2x - 6}{3} = \frac{2x}{3} - 2$

      I. $m = \frac{2}{3}$
      
      II. Any line perpendicular to the given line will have a gradient of $m = \frac{-3}{2}$, finding the equation of the line using that gradient and \((4,7)\)

         $$y = \frac{-3x}{2} + 13$$

   b. .. 
      
      $4^2 - k = 11$

      I. $k = 5$

      $$f(x) = x^2 - 5$$

      II. $f(3) = 4$

      III. $x^2 - 5 = 95$

      $x = 10$
6. Statistics
   a. Copy and complete the table

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales in $Thousands</td>
<td>38</td>
<td>35</td>
<td>27</td>
<td>15</td>
<td>10</td>
</tr>
</tbody>
</table>

   b. Greatest decrease is seen between March – April, the line is steepest there, or just find the gradient to confirm
   c. $25000 average sales
   d. June has $25,000
   e. Sales in June saw in increase

7. Transformations
   a. The coordinates \( R(2, 4), R'(2, 0) \)
   b. The transformation is a reflection in the line \( y = 2 \)
   c. Enlargement
      I. Draw the diagram
II. \( R''S''T'' = 3^2 \times 4 = 36 \text{ unit}^2 \)

III. Geometric relationships
   a. The shapes are similar
   b. \( R''S''T'' = 9RST \)

8. Investigation
   a. Draw the diagrams and complete the table

   ![Diagram of rectangles A, B, and C]

   b. Complete the table

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>Length</th>
<th>Width</th>
<th>Area</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>2</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>B</td>
<td>9</td>
<td>3</td>
<td>27</td>
<td>24</td>
</tr>
<tr>
<td>C</td>
<td>8</td>
<td>4</td>
<td>32</td>
<td>24</td>
</tr>
<tr>
<td>D</td>
<td>7</td>
<td>5</td>
<td>35</td>
<td>24</td>
</tr>
<tr>
<td>E</td>
<td>6</td>
<td>6</td>
<td>36</td>
<td>24</td>
</tr>
</tbody>
</table>

   Answers to parts b and c are in the table
9. Functions and relations
   a. 
      I. \( f(5) = \frac{3}{5} \)
      II. \( f^{-1}(x) = \frac{7}{2-x} \)
      III. \( gf(x) = \sqrt{\frac{2x-7}{x}} + 3 \)
   b. Completing the square
      1. \( 1 - 6x - x^2 = 10 - (x+3)^2 \)
         or \(-1(x+3)^2 + 10\)
      II. 
         a. Maximum value is \( y = 10 \)
         b. Axis of symmetry is \( x = -3 \)
         c. \( 1 - 6x - x^2 = 0 \)
            \( x = -6.16, x = 0.16 \)
10. Geometry and trigonometry

a. 
   I. OGF = 31°, OGF is an isosceles triangle
   II. DEF is supplementary to DGF, supplementary angles in a cyclic quadrilateral DEFG. DGF = DGO+OGF = 31+25=56 so DEF = 180 – 56 = 124°

b. Bearings
   I. Draw the diagram

   ![](image)

   II. Calculate
      a. ∠JKL = 90 + 54 = 144°
      b. JL, using the cosine rule JL = 174.1 km
      c. We need to find angle “a” add it to 90 and subtract that sum from 360, so using the sine rule to find angle a we get 24.3°, the bearing then is 360 – (90 + 24.3) = 245.7°,
11. Vectors and matrices

a. Matrices and transformations

I. Set up the matrix equation, find and use its inverse to give

\[
\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 3 & 5 \\ 7 & 2 \end{pmatrix} = \begin{pmatrix} 5 & -3 \\ 2 & 7 \end{pmatrix}
\]

\[
\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 7 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 5 & -3 \\ 2 & 7 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}
\]

II. Notice what the matrix \( \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \) does to the points, it switches them around and changes the sign of the y value so \( (x, y) = (1, -5) \)

III. The matrix \( M \) represents a 90 degree rotation about the origin

b. Vectors

I. ..

a. \( OP = \begin{pmatrix} 2 \\ 7 \end{pmatrix} \)

\[ OR = OP + PR \]

b. \[ OR = \begin{pmatrix} 2 \\ 7 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \]

II. ..

\[ RS = RO + OS \]

a. \[ RS = \begin{pmatrix} -6 \\ -4 \end{pmatrix} + \begin{pmatrix} 14 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \end{pmatrix} \]

b. Note that \( RS = 2PR \) this indicates that they are parallel to each other, also both vectors share the point \( R \), therefore they form a straight line \( RST \) and so \( R, S \) and \( T \) are collinear