FILL IN ALL THE INFORMATION REQUESTED CLEARLY IN CAPITAL LETTERS.

TEST CODE 01234020

SUBJECT MATHEMATICS – Paper 02

PROFICIENCY GENERAL

REGISTRATION NUMBER

SCHOOL/CENTRE NUMBER

NAME OF SCHOOL/CENTRE

CANDIDATE’S FULL NAME (FIRST, MIDDLE, LAST)

DATE OF BIRTH DDMMYYYY

SIGNATURE ____________________________

0123402001
READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of TWO sections: I and II.

2. Section I has EIGHT questions and Section II has THREE questions.

3. Answer ALL questions in Section I and any TWO questions from Section II.

4. Write your answers in the booklet provided.

5. Do NOT write in the margins.

6. All working MUST be clearly shown.

7. A list of formulae is provided on page 4 of this booklet.

8. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. Remember to draw a line through your original answer.

9. If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.

Required Examination Materials

Electronic calculator
Geometry set

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.
LIST OF FORMULAE

Volume of a prism \( V = Ah \) where \( A \) is the area of a cross section and \( h \) is the perpendicular length.

Volume of cylinder \( V = \pi r^2 h \) where \( r \) is the radius of the base and \( h \) is the perpendicular height.

Volume of a right pyramid \( V = \frac{1}{3} Ah \) where \( A \) is the area of the base and \( h \) is the perpendicular height.

Circumference \( C = 2\pi r \) where \( r \) is the radius of the circle.

Arc length \( S = \frac{\theta}{360} \times 2\pi r \) where \( \theta \) is the angle subtended by the arc, measured in degrees.

Area of a circle \( A = \pi r^2 \) where \( r \) is the radius of the circle.

Area of a sector \( A = \frac{\theta}{360} \times \pi r^2 \) where \( \theta \) is the angle of the sector, measured in degrees.

Area of trapezium \( A = \frac{1}{2} (a + b) h \) where \( a \) and \( b \) are the lengths of the parallel sides and \( h \) is the perpendicular distance between the parallel sides.

Roots of quadratic equations If \( ax^2 + bx + c = 0 \),

then \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Trigonometric ratios

\[ \sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} \]

\[ \cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \]

\[ \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} \]

Area of triangle Area of \( \triangle ABC \) = \( \frac{1}{2} \) \( bh \) where \( b \) is the length of the base and \( h \) is the perpendicular height.

Area of \( \triangle ABC \) = \( \frac{1}{2} \) \( ab \sin C \)

Area of \( \triangle ABC \) = \( \sqrt{s (s-a) (s-b) (s-c)} \)

where \( s = \frac{a+b+c}{2} \)

Sine rule \( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \)

Cosine rule \( a^2 = b^2 + c^2 - 2bc \cos A \)
SECTION I

Answer ALL questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the EXACT value of

\[
(3.6 + \sqrt{51.84}) \div 3.75
\]

(2 marks)

(b) The diagram below, not drawn to scale, shows two jars of peanut butter of the same brand.

Which of the jars shown above is the BETTER buy?
Show ALL working to support your answer.

(3 marks)
Thomas invested $1498 at 6% simple interest per annum.

Calculate:

(i) The interest, in dollars, earned after six months

(ii) The TOTAL amount of money in his account after 3 years

(iii) How long it will be before his investment earns $449.40

Total 11 marks
2.  

(a)  
(i)  Solve for $x$, where $x$ is a real number.

\[ 8 - x \leq 5x + 2 \]

(ii) Show your solution to (a) (i) on the number line below.

(b)  
Expand and simplify

\[ 2x (x + 5) - 3(x - 4) \]
(c) Simplify
\[ \frac{3x^2 \times 4x^3}{2x} . \]

(2 marks)

(d) Write as a single fraction, in its lowest terms,
\[ \frac{x + 1}{2} + \frac{5 - x}{5} . \]

(2 marks)

(e) Factorize completely
\[ 4x^2 - 4 \]

(2 marks)

Total 12 marks
3. (a) The Venn diagram below shows the number of students in Form 5A who have visited Canada (C) or Dominica (D).

```
\begin{center}
\begin{tikzpicture}
\node (A) at (0,0) {C};
\node (B) at (3,0) {D};
\node (U) at (4,0) {U};
\node (3) at (1.5,0) {3};
\node (x) at (2.5,0) {x};
\node (10) at (4,0) {10};
\node (2x) at (5.5,0) {2x};
\end{tikzpicture}
\end{center}
```

(i) How many students have visited Dominica ONLY?
.............................................................................................................................
(1 mark)

(ii) Write an expression, in terms of \( x \), to represent the TOTAL number of students who have visited Canada.
.............................................................................................................................
(1 mark)

(iii) Given that there are 25 students in Form 5A, calculate the value of \( x \).

..............................................................................................................................
(2 marks)

(iv) Hence, write down the number of students in each of the following subsets:

- \( C \cup D \) ..............................................................................................................

- \( C \cap D \) ..............................................................................................................

- \( (C \cup D)' \) ...........................................................................................................

(3 marks)
(b) (i) Using a ruler, a pencil and a pair of compasses, construct accurately, the square EFGH where EF = 6 cm.

(Show ALL construction lines and curves.)

(4 marks)

(ii) Measure, and state in centimetres, the length of the diagonal FH.

FH = ............................................................................................................. cm

(1 mark)

Total 12 marks
NOTHING HAS BEEN OMITTED.
4. (a) The diagram below shows a map of an island drawn on a grid of 1 cm squares.

(i) State, in cm, the length of LM as shown in the diagram.

\[ \text{LM} = \text{........................................................................................................... cm} \]

(1 mark)

(ii) Estimate, by counting squares, the area of the map shown in the diagram.

(1 mark)
(iii) On the island, the actual distance LM is 20 km. Complete the following statement:

On the map, 1 cm represents .............................................. km.

(1 mark)

(iv) Write the scale of the map in the form $1 : x$.

(1 mark)

(v) What distance on the island will be 3 cm on the map?

(1 mark)

(vi) What area on the island will be represented by 3 cm$^2$ on the map?

(2 marks)
(b) The diagram below, \textbf{not drawn to scale}, shows a prism with cross section PQRST and length 20 cm. PQRST is made up of a rectangle PQRT and a semicircle RST such that PQ = 6 cm and QR = 5 cm.

Use $\pi = 3.14$

(i) Calculate the area of the cross section PQRST.

(ii) An engineer needs a similar prism whose volume is NOT more than 900 cm$^3$. Estimate, in cm, the length of the longest prism he can use.
5. (a) In the diagram below, not drawn to scale, ST = 6 m, WR = 11.2 m, WT = 14.8 m and angle WRS = 90°.

Calculate, giving your answer to 1 decimal place

(i) the length RS

(ii) the measure of angle RTW.
(b) The graph below shows a triangle ABC and its image A'B'C' after undergoing a single transformation.
(i) Write down the coordinates of the vertices of \( \triangle ABC \).

\( \text{(1 mark)} \)

(ii) Write down the coordinates of the vertices of \( \triangle A'B'C' \).

\( \text{(1 mark)} \)

(iii) Describe FULLY the transformation that maps triangle \( \triangle ABC \) onto triangle \( \triangle A'B'C' \).

........................................................................................................................................
........................................................................................................................................
........................................................................................................................................

\( \text{(2 marks)} \)

(iv) On the graph on page 16, draw the line \( x = 1 \) AND the triangle \( \triangle A''B''C'' \), the image of triangle \( \triangle ABC \) after a reflection in the line \( x = 1 \).

\( \text{(3 marks)} \)

(v) State ONE geometrical relationship among \( \triangle ABC \), \( \triangle A'B'C' \) and \( \triangle A''B''C'' \).

........................................................................................................................................
........................................................................................................................................
........................................................................................................................................

\( \text{(1 mark)} \)

Total 12 marks
6. (a) The table below gives the number of cars sold in a country, in hundreds, from 2010 to 2014.

<table>
<thead>
<tr>
<th>Year</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cars sold (in hundreds)</td>
<td>19</td>
<td>10</td>
<td>26</td>
<td>16</td>
<td>30</td>
</tr>
</tbody>
</table>
(i) Complete the line graph on page 18 to represent the given information.  
\[(1 \text{ mark})\]

(ii) Between which two consecutive years was there the GREATEST increase in cars sold?  
\[\text{.............................................................................................................................} \]  
\[(1 \text{ mark})\]

(iii) What was the TOTAL number of cars sold in the five year period 2010 to 2014?  
\[(2 \text{ marks})\]

(iv) The mean number of cars sold from 2010 to 2015 was 22.5 hundred. How many cars were sold in 2015?  
\[(2 \text{ marks})\]
(b) (i) A line JK has equation $2y = 5x + 6$. Determine the gradient of JK.

Gradient of the line JK is ................................................................. .

(2 marks)

Another line GH, is perpendicular to JK and passes through the point (5, -1).

(ii) State the gradient of the line GH.

Gradient of the line GH is ................................................................. .

(1 mark)

(iii) Determine the equation of line GH.

Equation of the line GH is ................................................................. .

(2 marks)

Total 11 marks
NOTHING HAS BEEN OMITTED.
7. The table below shows how the minutes taken by all students to complete a science experiment were recorded and grouped.

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Number of Students who Completed (Frequency)</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6–10</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>11–15</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>16–20</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>21–25</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>26–30</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>31–35</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>36–40</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

(a) Complete the cumulative frequency column in the table. (2 marks)

(b) On the grid on page 23, using a scale of 2 cm to represent 5 minutes on the x-axis and 2 cm to represent 5 students on the y-axis, draw a cumulative frequency curve to represent the information in the table. (5 marks)

Using the graph, estimate

(c) (i) the median time taken to complete the experiment (2 marks)

(ii) the probability that a student, chosen at random, took 30 minutes or less to complete the experiment. (2 marks)

Show on your graph, using broken lines, how these estimates were obtained.

Total 11 marks
8. The diagram below shows the first three figures in a sequence of figures.

Figure 1

Figure 2

Figure 3

(a) Draw the fourth figure in the sequence.

(2 marks)
(b) The table below shows the number of dots and lines in each figure. Study the pattern in the table and complete the table by inserting the missing values in the rows numbered (i), (ii), (iii) and (iv).

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Dots</th>
<th>Number of Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>(i)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>\cdots</td>
<td>\cdots</td>
</tr>
<tr>
<td>(ii)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>\cdots</td>
<td>\cdots</td>
</tr>
<tr>
<td>(iii)</td>
<td>\cdots</td>
<td>49</td>
</tr>
<tr>
<td>(iv)</td>
<td>\cdots</td>
<td>\cdots</td>
</tr>
</tbody>
</table>

Entries omitted for Figures 5–9

Entries omitted for some Figures

Entries omitted for some Figures

Total 10 marks
9. (a) The diagram below shows the graph of three lines and a shaded region, $S$, defined by three inequalities associated with these lines.

The inequality associated with the line $y = 3$ is $y \geq 3$.

\[ y \geq 3 \]

(i) State the other TWO inequalities which define the shaded region.

...................................................................................................................................

(2 marks)
The function $P = 5x + 2y - 3$ satisfies the solution set represented by the closed triangular region.

(ii) Identify the three pairs of $(x, y)$ values for which $P$ has a maximum or a minimum value.

....................................................................................................................................
....................................................................................................................................
....................................................................................................................................

(2 marks)

(iii) Which pair of $(x, y)$ values makes $P$ a maximum?

Justify your answer.

P is a maximum at ................................................................. .

(3 marks)
(b) The function \( f(x) \) and \( g(x) \) are defined as follows

\[
f(x) = \frac{3}{2x + 1} \quad \text{and} \quad g(x) = x^2
\]

(i) Evaluate EACH of the following:

\[
\begin{align*}
\cdot & \quad g\left(\frac{-1}{2}\right) \\
\cdot & \quad f g\left(\frac{-1}{2}\right)
\end{align*}
\]

(ii) Write an expression in \( x \) for \( f^{-1}(x) \).

(4 marks)

Total 15 marks
10. (a) The figure below, not drawn to scale, shows a circle with centre O. The radius of the circle is 21 cm and angle HOK = 40°.

Determine

(i) the area of the minor sector HOK

(ii) the area of triangle HOK

(iii) the area of the shaded segment.
The diagram below, not drawn to scale, shows a circle with centre \( O \). \( TAE \) is a tangent to the circle at point \( A \) and angle \( AOD = 72^\circ \).

Calculate, giving the reason for each step of your answer, the measure of:

(i) \( \angle ADC = \) ............................................

(2 marks)

(ii) \( \angle ACD = \) ............................................

(2 marks)

(iii) \( \angle CAD = \) ............................................

(2 marks)

(iv) \( \angle OEA \quad \) ............................................

(2 marks)

Total 15 marks
VECTORS AND MATRICES

11. (a) The points A, B and C have coordinates A (–2,8), B (4,2) and C (0,9). M is the midpoint of the line segment AB.

(i) Express EACH of the following in the form \(\begin{pmatrix} x \\ y \end{pmatrix} \):

\[\text{• } \overrightarrow{OB} = \text{.................................} \]

\[\text{• } \overrightarrow{AB} = \text{.................................} \]

\[\text{• } \overrightarrow{OM} = \text{.................................} \]

(5 marks)

(ii) Using a vector method, show that \(\overrightarrow{AC}\) and \(\overrightarrow{OB}\) are parallel.

(2 marks)
(b) The matrix M is defined as
\[
M = \begin{pmatrix} 2p & -3 \\ 4 & 1 \end{pmatrix}.
\]
Determine the value of \( p \) for which the matrix M is singular.

(2 marks)

(c) A and B are two \( 2 \times 2 \) matrices such that
\[
A = \begin{pmatrix} 1 & 2 \\ -4 & 3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 5 & -1 \\ 0 & 3 \end{pmatrix}.
\]

(i) Calculate \( 2A + B \).

(2 marks)

(ii) Determine \( B^{-1} \), the inverse of \( B \).

(2 marks)
(iii) Given that \[
\begin{pmatrix}
5 & -1 \\
0 & 3
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix} = \begin{pmatrix}
9 \\
3
\end{pmatrix},
\]
calculate the values of \(x\) and \(y\).

(2 marks)

Total 15 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.
EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No. 

01234020/JANUARY/F 2016
CANDIDATE’S RECEIPT

INSTRUCTIONS TO CANDIDATE:

1. Fill in all the information requested clearly in capital letters.

   TEST CODE 0123402036

   SUBJECT: MATHEMATICS

   PROFICIENCY: GENERAL

   FULL NAME: ____________________________
   (BLOCK LETTERS)

   Signature: ____________________________

   Date: ____________________________

2. Ensure that this slip is detached by the Supervisor or Invigilator and given to you when you hand in this booklet.

3. Keep it in a safe place until you have received your results.

INSTRUCTION TO SUPERVISOR/INVIGILATOR:

Sign the declaration below, detach this slip and hand it to the candidate as his/her receipt for this booklet collected by you.

I hereby acknowledge receipt of the candidate’s booklet for the examination stated above.

Signature: ____________________________

                   Supervisor/Invigilator

Date: ____________________________