CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

04 JANUARY 2019 (a.m.)

FILL IN ALL THE INFORMATION REQUESTED CLEARLY IN CAPITAL LETTERS.

TEST CODE 0 1 2 3 4 0 2 0

SUBJECT MATHEMATICS – Paper 02

PROFICIENCY GENERAL

REGISTRATION NUMBER

SCHOOL/CENTRE NUMBER

NAME OF SCHOOL/CENTRE

CANDIDATE’S FULL NAME (FIRST, MIDDLE, LAST)

DATE OF BIRTH D D M M Y Y Y Y

SIGNATURE ____________________________________________
READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of TWO sections: I and II.

2. Section I has SEVEN questions and Section II has THREE questions.

3. Answer ALL questions.

4. Write your answers in the spaces provided in this booklet.

5. Do NOT write in the margins.

6. All working MUST be clearly shown.

7. A list of formulae is provided on page 4 of this booklet.

8. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**

9. If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.

**Required Examination Materials**

Electronic calculator
Geometry set

**DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.**
LIST OF FORMULAE

Volume of a prism \[ V = Ah \] where \( A \) is the area of a cross-section and \( h \) is the perpendicular length.

Volume of a cylinder \[ V = \pi r^2 h \] where \( r \) is the radius of the base and \( h \) is the perpendicular height.

Volume of a right pyramid \[ V = \frac{1}{3} Ah \] where \( A \) is the area of the base and \( h \) is the perpendicular height.

Circumference \[ C = 2\pi r \] where \( r \) is the radius of the circle.

Arc length \[ S = \frac{\theta}{360} \times 2\pi r \] where \( \theta \) is the angle subtended by the arc, measured in degrees.

Area of a circle \[ A = \pi r^2 \] where \( r \) is the radius of the circle.

Area of a sector \[ A = \frac{\theta}{360} \times \pi r^2 \] where \( \theta \) is the angle of the sector, measured in degrees.

Area of a trapezium \[ A = \frac{1}{2} (a + b) h \] where \( a \) and \( b \) are the lengths of the parallel sides and \( h \) is the perpendicular distance between the parallel sides.

Roots of quadratic equations If \( ax^2 + bx + c = 0 \),

then \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Trigonometric ratios

\[ \sin \theta = \frac{\text{length of opposite side}}{\text{length of hypotenuse}} \]
\[ \cos \theta = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}} \]
\[ \tan \theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}} \]

Area of a triangle

Area of \( \triangle \) \[ \frac{1}{2} bh \] where \( b \) is the length of the base and \( h \) is the perpendicular height.

Area of \( \triangle ABC \) \[ = \frac{1}{2} ab \sin C \]

Area of \( \triangle ABC \) \[ = \sqrt{s} (s-a) (s-b) (s-c) \]

where \( s = \frac{a+b+c}{2} \)

Sine rule \[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

Cosine rule \[ a^2 = b^2 + c^2 - 2bc \cos A \]
SECTION I

Answer ALL questions.

All working must be clearly shown.

1. (a) Evaluate

   (i) $3.8 \times 10^2 + 1.7 \times 10^3$, giving your answer in standard form

   ........................................................................................................................................

   (2 marks)

   (ii) $\frac{1}{2} \times \frac{3}{5}$, giving your answer as a fraction in its lowest terms.

   ........................................................................................................................................

   (2 marks)
(b) Express the number 6 as a binary number.

.......................................................................................................................................................

(1 mark)

(c) John bought a car for $65 000. If the value of the car depreciates by 8% each year, how much will the car be worth at the end of 2 years?

.......................................................................................................................................................

(2 marks)
(d) The table below shows the results obtained by a student in her CSEC Mathematics examination. The maximum mark for each paper is given in the third column of the table.

<table>
<thead>
<tr>
<th>Paper</th>
<th>Percentage Obtained</th>
<th>Maximum Mark for Paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>55</td>
<td>30</td>
</tr>
<tr>
<td>02</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>03</td>
<td>80</td>
<td>20</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

Determine, as a percentage, the student’s final mark for the Mathematics examination.

.......................................................................................................................................................
.......................................................................................................................................................

(2 marks)

Total 9 marks
2. (a) (i) Make \( x \) the subject of the formula

\[
y = \frac{x}{5} + 3p.
\]

(ii) Solve the following equation by factorization.

\[
2x^2 - 9x = 0
\]
(b) A farmer wishes to enclose a rectangular plot with a wire fence. The width of the plot is 3 metres less than its length, \( l \).

Given that the area enclosed by the fence is 378 square metres, show that

\[ l^2 - 3l - 378 = 0. \]

(2 marks)
(c) The force, $F$, applied to an object is directly proportional to the extension, $e$, produced by that object.

(i) Represent this information as an equation in terms of $F$, $e$ and an appropriate constant, $k$.

(ii) The incomplete table below shows corresponding values of $F$ and $e$.

<table>
<thead>
<tr>
<th>$F$</th>
<th>8</th>
<th>25</th>
<th>60</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>0.2</td>
<td>$x$</td>
<td>1.5</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Using the equation obtained in (c) (i), or otherwise, determine the value of $x$ and $y$.

---

Total 9 marks
3. (a) Using a ruler, a pencil and a pair of compasses, construct the right-angled triangle $ABC$, such that $AB = 5$ cm, $\angle ABC = 90^\circ$ and $\angle BAC = 60^\circ$.

(4 marks)
(b) The diagram below shows a right-angled triangle with sides \(a\) units, \(b\) units and \(c\) units.

(i) Using the diagram,

a) express \(c\) in terms of \(a\) and \(b\)

..............................................................................................................................

(1 mark)

b) write, in terms of \(a\), \(b\) and \(c\), an expression for \(\sin \theta + \cos \theta\).

..............................................................................................................................

(2 marks)
(ii) Using the results from (i) a) and b) on page 12, show that \((\sin \theta)^2 + (\cos \theta)^2 = 1\).

........................................................................................................................................

(2 marks)

Total 9 marks
4. (a) Given the function \( h(x) = \frac{2x + 3}{5 - x} \), determine

(i) the value of \( x \) for which the function is undefined

........................................................................................................................................ (1 mark)

(ii) an expression for \( h^{-1}(x) \).

........................................................................................................................................ (3 marks)
(b) The graph below shows a straight line intersecting the $x$ and $y$ axes.

Using the graph, determine the

(i) gradient of the line

(2 marks)
(ii) equation of the line

........................................................................................................................................

(1 mark)

(iii) equation of the perpendicular line that passes through $P$.

........................................................................................................................................

(2 marks)

Total 9 marks
5. (a) A survey was conducted among 48 persons to find out what mobile network they used. The table below shows the results of the survey.

<table>
<thead>
<tr>
<th>Mobile Network</th>
<th>WireTech</th>
<th>DigiLec</th>
<th>O-Fone</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Persons</td>
<td>20</td>
<td>12</td>
<td>10</td>
<td>6</td>
</tr>
</tbody>
</table>

(i) If this information is to be represented on a pie chart, what is the angle for the sector that will represent O-Fone?

(ii) Using the circle below, with radius shown, represent the information in the table above on a clearly labelled pie chart.

(2 marks)

(3 marks)
(b) The incomplete table below shows the results obtained by 200 boys and 250 girls on a Spanish examination.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Boys</th>
<th>Girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>62</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>200</td>
<td>250</td>
</tr>
</tbody>
</table>

Standard deviation 8.2 6.3

(i) A girl is chosen at random. Determine the probability that she achieves a Grade I.

........................................................................................................................................
(1 mark)
(ii) What percentage of the boys who took the exam achieved Grades I to III?

........................................................................................................................................

(2 marks)

(iii) Considering the standard deviations in the table on page 18, compare the performance of the boys and the girls.

........................................................................................................................................

(1 mark)

Total 9 marks
6. The diagram below, not drawn to scale, shows an open cylindrical container made of metal with a circular base and uniform thickness throughout. The length of the container, from the top to outer bottom, is 120 cm and the inner and outer radii are 14 cm and 15 cm respectively.

Take $\pi$ to be $\frac{22}{7}$.

(a) Draw a cross-sectional view of the container showing the measurements of the inner and outer radii.
(b) Show that the capacity of the container is 73 304 cm$^3$.

.......................................................................................................................................................
(2 marks)

(c) Determine the volume of the material used to make the container.

.......................................................................................................................................................
(3 marks)
(d) Given that the density of the material used to make the container is 2.2 g/cm$^3$, determine the mass, in kg, of the empty container.

\[
\text{density} = \frac{\text{mass}}{\text{volume}}
\]

(2 marks)

Total 9 marks
7. A sequence of figures is made from joining polygons with sides of unit length. The first three figures in the sequence are shown below.

(a) Draw Figure 4 of the sequence.
(b) Study the pattern of numbers in each row of the table below. Each row relates to one of the figures in the sequence of figures on page 23. Some rows have not been included in the table. Complete the rows numbered (i), (ii) and (iii).

<table>
<thead>
<tr>
<th>Figure 1</th>
<th>Number of Outer Lines of Unit Length</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1 + 2 + 2$</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>$2 + 2 + 4$</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>$3 + 2 + 6$</td>
<td>11</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>i</td>
<td></td>
<td>(2 marks)</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>ii</td>
<td></td>
<td>65</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>iii</td>
<td></td>
<td>(2 marks)</td>
</tr>
<tr>
<td>$n$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) Show that no figure can have a perimeter of 100 units.
SECTION II

Answer ALL questions.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. (a) (i) Complete the table below for the function \( f(x) = 3 + 2x - x^2 \).

\[
\begin{array}{c|c|c|c|c|c|c|c}
 x & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\
 f(x) & 0 & 3 & 4 & 0 & -5 \\
\end{array}
\]

(1 mark)
(ii) Complete the grid below to show all the points in the table on page 25 and hence, draw the graph of the function \( f(x) = 3 + 2x - x^2 \) for \(-2 \leq x \leq 4\).
(iii) Using the graph on page 26, determine

a) the coordinates of the maximum point of \( f(x) \)

b) the range of values of \( x \) for which \( f(x) > 0 \)

c) the gradient of \( f(x) \) at \( x = 1 \).
(b) Mr Thomas makes $x$ bottles of juice and $y$ cakes each day. To supply his customers, he makes at least 20 bottles of juice and no more than 15 cakes each day.

(i) Write TWO inequalities to represent this information.

........................................................................................................................................

(ii) Each day, Mr Thomas uses $163 to make the bottles of juice and the cakes. The cost to make a bottle of juice is $3.50 while the cost to make a cake is $5.25.

Write an inequality to represent this information.

........................................................................................................................................

........................................................................................................................................

(2 marks)
(iii) Show that on any given day, it is NOT possible for Mr Thomas to make 50 bottles of juice and 12 cakes.

(1 mark)

Total 12 marks
GEOMETRY AND TRIGONOMETRY

9. (a) The diagram below, not drawn to scale, shows a circle. The points \( P, Q, R, T \) and \( V \) are on the circumference. \( QRS \) is a straight line. Angle \( PVR = 75^\circ \) and angle \( TRS = 60^\circ \).

Determine the value of EACH of the following angles. Show detailed working where necessary and give a reason to support your answer.

(i) Angle \( PTR \)

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........................................................................................................................................
........................................................................................................................................

(2 marks)
(ii) Angle $TPQ$

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........................................................................................................................................
........................................................................................................................................

(2 marks)

(iii) Obtuse angle $POR$ where $O$ is the centre of the circle.

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........................................................................................................................................
........................................................................................................................................

(2 marks)
(b) A person at the top of a lighthouse, $TB$, sees two ships, $S_1$ and $S_2$, approaching the coast as illustrated in the diagram below. The angles of depression are $12^\circ$ and $20^\circ$ respectively. The ships are 110 m apart.

(i) Complete the diagram below by inserting the angles of depression and the distance between the ships.

(ii) Determine, to the nearest metre,

a) the distance, $TS_2$, between the top of the lighthouse and Ship 2
b) the height of the lighthouse, \( TB \).

(2 marks)

Total 12 marks
VECTORS AND MATRICES

10. (a) Three matrices are given as follows:

\[
P = \begin{pmatrix} -1 & 2 \\ 0 & 5 \end{pmatrix}, \quad Q = \begin{pmatrix} a \\ b \end{pmatrix} \text{ and } R = \begin{pmatrix} 11 \\ 15 \end{pmatrix}.
\]

(i) Using a calculation to support your answer, explain whether matrix \( P \) is a singular or a non-singular matrix.

........................................................................................................................................

(2 marks)
(ii) Given that $PQ = R$, determine the values of $a$ and $b$.

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..................................................................................................................................................................................
..................................................................................................................................................................................
..................................................................................................................................................................................
..................................................................................................................................................................................
..................................................................................................................................................................................
..................................................................................................................................................................................
..................................................................................................................................................................................

(3 marks)

(iii) State the reason why the matrix product $QP$ is not possible.

..................................................................................................................................................................................
..................................................................................................................................................................................
..................................................................................................................................................................................
..................................................................................................................................................................................

(1 mark)
(b) \( OABC \) is a parallelogram. \( X \) is the midpoint of \( AB \) and \( Y \) is the midpoint of \( BC \).
\( \overrightarrow{OA} = r \) and \( \overrightarrow{OC} = s \).

(i) Complete the diagram below to represent ALL the information given above.

(ii) Given that \( \overrightarrow{OX} + \overrightarrow{OY} = k \ (r + s) \), where \( k \) is a constant, using a vector method, find the value of \( k \).
EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No.
EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No. [ ]
INSTRUCTIONS TO CANDIDATE:

1. Fill in all the information requested clearly in capital letters.

TEST CODE: 0 1 2 3 4 0 2 0

SUBJECT: MATHEMATICS – Paper 02

PROFICIENCY: GENERAL

REGISTRATION NUMBER: [ ] [ ] [ ] [ ] [ ] [ ] [ ]

FULL NAME: ____________________________________________ (BLOCK LETTERS)

Signature: ________________________________________________

Date: ____________________________________________________

2. Ensure that this slip is detached by the Supervisor or Invigilator and given to you when you hand in this booklet.

3. Keep it in a safe place until you have received your results.

INSTRUCTION TO SUPERVISOR/INVIGILATOR:

Sign the declaration below, detach this slip and hand it to the candidate as his/her receipt for this booklet collected by you.

I hereby acknowledge receipt of the candidate’s booklet for the examination stated above.

Signature: _____________________________
Supervisor/Invigilator

Date: ________________________________