FILL IN ALL THE INFORMATION REQUESTED CLEARLY IN CAPITAL LETTERS.

TEST CODE 01234020

SUBJECT MATHEMATICS – Paper 02

PROFICIENCY GENERAL

REGISTRATION NUMBER

SCHOOL/CENTRE NUMBER

NAME OF SCHOOL/CENTRE

CANDIDATE’S FULL NAME (FIRST, MIDDLE, LAST)

DATE OF BIRTH D D M M Y Y Y

SIGNATURE __________________________
DO NOT WRITE ON THIS PAGE
READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of TWO sections: I and II.
2. Section I has SEVEN questions and Section II has THREE questions.
3. Answer ALL questions.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. All working MUST be clearly shown.
7. A list of formulae is provided on page 4 of this booklet.
8. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. Remember to draw a line through your original answer.
9. If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.

Required Examination Materials

Electronic calculator
Geometry set

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

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01234020/J/CSEC 2020
**LIST OF FORMULAE**

Volume of a prism

\[ V = Ah \] where \( A \) is the area of a cross-section and \( h \) is the perpendicular length.

Volume of a cylinder

\[ V = \pi r^2 h \] where \( r \) is the radius of the base and \( h \) is the perpendicular height.

Volume of a right pyramid

\[ V = \frac{1}{3} Ah \] where \( A \) is the area of the base and \( h \) is the perpendicular height.

Circumference

\[ C = 2\pi r \] where \( r \) is the radius of the circle.

Arc length

\[ S = \frac{\theta}{360} \times 2\pi r \] where \( \theta \) is the angle subtended by the arc, measured in degrees.

Area of a circle

\[ A = \pi r^2 \] where \( r \) is the radius of the circle.

Area of a sector

\[ A = \frac{\theta}{360} \times \pi r^2 \] where \( \theta \) is the angle of the sector, measured in degrees.

Area of a trapezium

\[ A = \frac{1}{2} (a + b) h \] where \( a \) and \( b \) are the lengths of the parallel sides and \( h \) is the perpendicular distance between the parallel sides.

Roots of quadratic equations

If \( ax^2 + bx + c = 0 \),

then \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Trigonometric ratios

\[ \sin \theta = \frac{\text{length of opposite side}}{\text{length of hypotenuse}} \]

\[ \cos \theta = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}} \]

\[ \tan \theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}} \]

Area of a triangle

Area of \( \Delta = \frac{1}{2} bh \) where \( b \) is the length of the base and \( h \) is the perpendicular height.

Area of \( \Delta ABC = \frac{1}{2} ab \sin C \)

Area of \( \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} \)

where \( s = \frac{a+b+c}{2} \)

Sine rule

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

Cosine rule

\[ a^2 = b^2 + c^2 - 2bc \cos A \]
SECTION I

Answer ALL questions.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the EXACT value of the following:

   (i) \( \frac{4 \frac{1}{5}}{\frac{3}{3}} - 1 \frac{1}{4} \)

   \( \frac{4 \frac{1}{5}}{\frac{3}{3}} - 1 \frac{1}{4} \)

   (2 marks)

   (ii) \( \frac{4.1 - 1.25^2}{0.005} \)

   \( \frac{4.1 - 1.25^2}{0.005} \)

   (2 marks)

GO ON TO THE NEXT PAGE
(b) A stadium currently has a seating capacity of 15 400 seats.

(i) Calculate the number of people in the stadium when 75% of the seats are occupied.

(ii) The stadium is to be renovated with a new seating capacity of 20 790 seats. After the renovation, what will be the percentage increase in the number of seats?
(c) A neon light flashes five times every 10 seconds. Show that the light flashes 43,200 times in one day.

\[
\text{(2 marks)}
\]

Total 9 marks
2. (a) Factorize the following expressions completely.

(i) $5h^2 - 12hg$

(ii) $2x^2 - x - 6$

(1 mark)

(2 marks)
(b) Solve the equation

\[ r + 3 = 3(r - 5). \]

(c) Make \( k \) the subject of the formula

\[ 2A = \pi k^2 + 3t. \]
(d) A farmer plants two crops, potatoes and corn, on a ten-hectare piece of land. The number of hectares of corn planted, \( c \), must be at least twice the number of hectares of potatoes, \( p \).

Write TWO inequalities to represent the scenario above.

\[ c \geq 2p \]

(2 marks)

Total 9 marks
3. (a) The diagram below shows a hexagonal prism.

Complete the following statement.

The prism has

............................................ faces,

............................................ edges and

............................................ vertices. (3 marks)
(b) A sports club owns a field $PQRS$, in the shape of a quadrilateral. A scale diagram of this field is shown below. (1 centimetre represents 10 metres.)

In the following parts, show all your construction lines.

The field is to be divided with a fence from $P$ to the side $RS$, so that different sports can be played at the same time.

Each point on the fence is the same distance from $PQ$ as from $PS$.

(i) Using a straight edge and compasses only, construct the line representing the fence. 

   (1 mark)

(ii) Write down the length of this fence, in metres.

   (1 mark)
(c) A quadrilateral $PQRS$ and its image $P'Q'R'S'$ are shown on the grid below.

(i) Write down the mathematical name for the quadrilateral $PQRS$.

.................................................................................................................................

(1 mark)

(ii) $PQRS$ is mapped onto $P'Q'R'S'$ by an enlargement with scale factor, $k$, about centre, $C(a, b)$. Using the diagram above, determine the values of $a$, $b$ and $k$.

.................................................................................................................................

.................................................................................................................................

(3 marks)

Total 9 marks
4. (a) The function $f$ is defined as

$$f(x) = \frac{2x + 7}{5}.$$ 

(i) Find the value of $f(4) + f(-4)$. 

(2 marks)
(ii) a) Calculate the value of \( x \) for which \( f(x) = 9 \).

(2 marks)

b) Hence or otherwise, determine the value of \( f^{-1}(9) \).

(1 mark)
(b) The graph below shows two straight lines, $L_1$ and $L_2$. $L_1$ intercepts the $x$ and $y$ axes at $(4, 0)$ and $(0, 2)$ respectively. $L_2$ intercepts the $x$ and $y$ axes at $(1.5, 0)$ and $(0, -3)$ respectively.

(i) Determine the equation of the line $L_1$. 

(3 marks)
(ii) What is the gradient of the line $L_2$, given that $L_1$ and $L_2$ are perpendicular?

(1 mark)

Total 9 marks
5. A group of 100 students estimated the mass, \( m \) (grams), of a seed. The cumulative frequency curve below shows the results.

\[ \begin{array}{|c|c|}
\hline
\text{Mass of seed (grams)} & \text{Cumulative frequency (number of students)} \\
\hline
0 & 0 \\
1 & 10 \\
2 & 20 \\
3 & 30 \\
4 & 40 \\
5 & 50 \\
6 & 60 \\
7 & 70 \\
8 & 80 \\
9 & 90 \\
10 & 100 \\
11 & 100 \\
\hline
\end{array} \]

(a) Using the cumulative frequency curve, estimate the

(i) median

(1 mark)
(ii) upper quartile

(1 mark)

(iii) semi-interquartile range

(2 marks)

(iv) number of students whose estimate is 2.8 grams or less.

(1 mark)
(b) 

(i) Use the cumulative frequency curve on page 18 to complete the frequency table below.

<table>
<thead>
<tr>
<th>Mass of Seed, $m$ (grams)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; m \leq 2$</td>
<td>20</td>
</tr>
<tr>
<td>$2 &lt; m \leq 4$</td>
<td></td>
</tr>
<tr>
<td>$4 &lt; m \leq 6$</td>
<td></td>
</tr>
<tr>
<td>$6 &lt; m \leq 8$</td>
<td>6</td>
</tr>
<tr>
<td>$8 &lt; m \leq 10$</td>
<td>1</td>
</tr>
</tbody>
</table>

(2 marks)

(ii) A student is chosen at random. Find the probability that the student estimated the mass to be greater than 6 grams.

(2 marks)

Total 9 marks
6. (a) The radius of EACH circle in the rectangle $WXYZ$ shown below is 7 cm. The circles fit \textbf{exactly} into the rectangle.

(i) Show that the area of the rectangle is 2352 cm$^2$.

(2 marks)
(ii) Calculate the area of the shaded region.

\[ \text{Take } \pi \text{ to be } \frac{22}{7}. \]
(b) The diagram below, not drawn to scale, shows triangle $MNP$ in which angle $MPN = \angle PMN = 52^\circ$ and $MN = 12.5$ cm.

(i) State the type of triangle shown above.

.................................................................................................................. (1 mark)

(ii) Determine the value of angle $PNM$.

.................................................................................................................. (1 mark)

(iii) Calculate the area of the triangle $MNP$.

.................................................................................................................. (2 marks)

Total 9 marks

GO ON TO THE NEXT PAGE
7. A sequence of figures is made up of stars, using dots and sticks of different lengths. The first three figures in the sequence are shown below.

![Figures 1, 2, and 3]

Study the pattern of numbers in each row of the table below. Each row relates to a figure in the sequence of figures started above. Some rows have not been included in the table.

(a) Complete Rows (i), (ii) and (iii).

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Sticks (S)</th>
<th>Number of Dots (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
<td>37</td>
</tr>
<tr>
<td>4</td>
<td>48</td>
<td>49</td>
</tr>
<tr>
<td>(i)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td></td>
<td>156</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(b) The sum of the number of dots in two consecutive figures are recorded. This information for the first three pairs of consecutive figures are shown in the table below.

<table>
<thead>
<tr>
<th>Figure 1 and Figure 2</th>
<th>Figure 2 and Figure 3</th>
<th>Figure 3 and Figure 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$13 + 25 = 38$</td>
<td>$25 + 37 = 62$</td>
<td>$37 + 49 = 86$</td>
</tr>
</tbody>
</table>

Determine the TOTAL number of dots in

(i) Figure 7 and Figure 8

(ii) Figure $n$ and Figure $(n + 1)$

(2 marks)

(2 marks)

Total 10 marks

GO ON TO THE NEXT PAGE
SECTION II

Answer ALL questions.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. (a) Solve the pair of simultaneous equations:

\[ y^2 + 2y + 11 = x \]

\[ x = 5 - 3y \]
(b) The function $f$ is defined as follows:

$$f(x) = 4x^2 - 8x - 2$$

(i) Express $f(x)$ in the form $a(x + h)^2 + k$, where $a$, $h$ and $k$ are constants.

(ii) State the minimum value of $f(x)$.

(iii) Determine the equation of the axis of symmetry.
(c) The speed–time graph below, **not drawn to scale**, shows the three-stage journey of a car over a period of 40 seconds.

Determine the acceleration of the car for EACH of the following stages of the journey.

Stage 2

Stage 3

(3 marks)

Total 12 marks
NOTHING HAS BEEN OMITTED.
9. (a) The circle shown below has centre $O$ and the points $A$, $B$, $C$ and $D$ lying on the circumference. A straight line passes through the points $A$ and $B$. Angle $CBD = 49^\circ$ and angle $OAB = 37^\circ$.

![Diagram of a circle with points A, B, C, and D, and angles 49°, 37°, x°, and y° at O.]

(i) Write down the mathematical names of the straight lines $BC$ and $OA$.

$BC$ .................................................................

$OA$ .................................................................

(2 marks)
(ii) Determine the value of EACH of the following angles. Show detailed working where necessary and give a reason to support your answer.

a) $x$

Reason

........................................................................................................................................
........................................................................................................................................
........................................................................................................................................
........................................................................................................................................
........................................................................................................................................
........................................................................................................................................

(2 marks)

b) $y$

Reason

........................................................................................................................................
........................................................................................................................................
........................................................................................................................................
........................................................................................................................................
........................................................................................................................................
........................................................................................................................................

(2 marks)
(b) The diagram below, **not drawn to scale**, shows the route of a ship cruising from Palmcity (P) to Quayton (Q) and then to Rivertown (R). The bearing of Q from P is 133° and the angle PQR is 56°.

(i) Calculate the value of angle W.

(2 marks)
(ii) Determine the bearing of $P$ from $Q$.

(1 mark)

(iii) Calculate the distance $RP$.

(3 marks)

Total 12 marks
10. (a) The transformation $M = \begin{bmatrix} 0 & p \\ q & 0 \end{bmatrix}$ maps the point $R$ onto $R'$ as shown in the diagram below.
(i) Determine the values of $p$ and $q$.

.........................................................................................................................

(2 marks)

(ii) Describe fully the transformation, $M$.

.........................................................................................................................

.........................................................................................................................

.........................................................................................................................

(3 marks)
(b) $PQRS$ is a parallelogram in which $\overrightarrow{PQ} = u$ and $\overrightarrow{PS} = v$.

$M$ is a point on $QS$ such that $QM : MS = 1 : 2$.

(i) Write in terms of $u$ and $v$ an expression for

a) $\overrightarrow{QS}$

(1 mark)

b) $\overrightarrow{QM}$.

(1 mark)
(ii) Show that $\vec{MR} = \frac{1}{3}(\vec{u} + 2\vec{v})$.

(2 marks)

(iii) $T$ is the mid-point of $PQ$. Prove that $R$, $M$ and $T$ are collinear.

(3 marks)

Total 12 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.
EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No.  

01234020/J/CSEC 2020
CANDIDATE'S RECEIPT

INSTRUCTIONS TO CANDIDATE:

1. Fill in all the information requested clearly in capital letters.

   TEST CODE: [01234020]
   SUBJECT: MATHEMATICS – Paper 02
   PROFICIENCY: GENERAL
   REGISTRATION NUMBER: 
   FULL NAME: ____________________________ (BLOCK LETTERS)
   Signature: ____________________________
   Date: ____________________________

2. Ensure that this slip is detached by the Supervisor or Invigilator and given to you when you hand in this booklet.

3. Keep it in a safe place until you have received your results.

INSTRUCTION TO SUPERVISOR/INVIGILATOR:

Sign the declaration below, detach this slip and hand it to the candidate as his/her receipt for this booklet collected by you.

I hereby acknowledge receipt of the candidate’s booklet for the examination stated above.

Signature: ____________________________
Supervisor/Invigilator

Date: ____________________________