

FORM TP 2005106

TEST CODE **01234020**

MAY/JUNE 2005

**CARIBBEAN EXAMINATIONS COUNCIL
SECONDARY EDUCATION CERTIFICATE
EXAMINATION
MATHEMATICS**

Paper 02 – General Proficiency

2 hours 40 minutes

26 MAY 2005 (a.m.)

INSTRUCTIONS TO CANDIDATES

1. Answer ALL questions in Section I, and ANY TWO in Section II.
2. Write your answers in the booklet provided.
3. All working must be shown clearly.
4. A list of formulae is provided on page 2 of this booklet.

Examination Materials

Electronic calculator (non-programmable)
Geometry set
Mathematical tables (provided)
Graph paper (provided)

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LIST OF FORMULAE

Volume of a prism $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of a right pyramid $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference $C = 2\pi r$ where r is the radius of the circle.

Area of a circle $A = \pi r^2$ where r is the radius of the circle.

Area of trapezium $A = \frac{1}{2}(a + b)h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

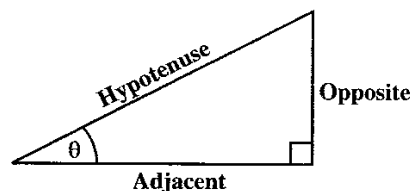
Roots of quadratic equations If $ax^2 + bx + c = 0$,
then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



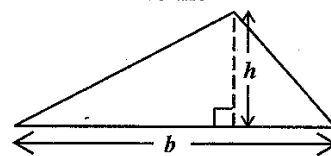
Area of triangle

Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height

$$\text{Area of } \Delta ABC = \frac{1}{2}ab \sin C$$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$

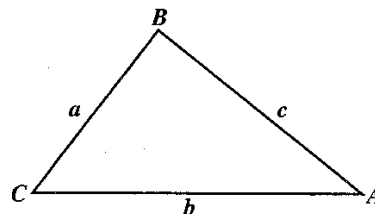


Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$



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SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. (a) Calculate the EXACT value of

$$4\frac{1}{5} - (1\frac{1}{9} \times 3)$$

(3 marks)

- (b) The table below shows Amanda's shopping bill. Some numbers were removed and replaced with letters

Items	Quantity	Unit Price (\$)	Total Cost (\$)
Stickers	12	0.49	5.88
T-shirts	3	12.50	A
CD's	2	B	33.90
Posters	C	6.20	31.00
Total			108.28
15% VAT (to the nearest cent)			D

- (i) Calculate the values of A, B, C and D. (5 marks)
- (ii) Amanda sold 6 of the 12 stickers which she had bought at 75 cents each, and the remaining stickers at 40 cents each.

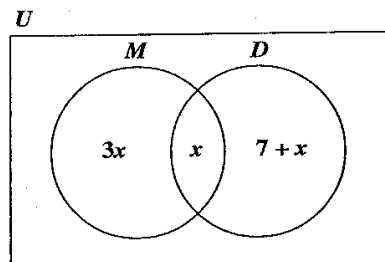
Show, using calculations, whether Amanda made a profit or loss on buying and selling stickers. (3 marks)

Total 11 marks

2. (a) Factorise
- (i) $5a^2b + ab^2$ (2 marks)
- (ii) $9k^2 - 1$ (2 marks)
- (iii) $2y^2 - 5y + 2$ (2 marks)
- (b) Expand and simplify
 $(2x + 5)(3x - 4)$ (2 marks)
- (c) Adam, Imran and Shakeel were playing a card game.
 Adam scored x points
 Imran scored 3 points fewer than Adam
 Shakeel scored twice as many points as Imran
 Together they scored 39 points.
- (i) Write down, in terms of x , an expression for the number of points scored by Shakeel. (2 marks)
- (ii) Write an equation which may be used to find the value of x . (2 marks)

Total 12 marks

3. (a)



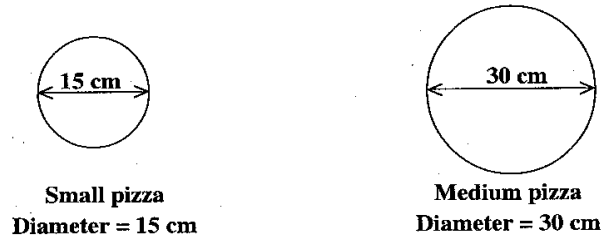
In the diagram shown above, the Universal set, (U), represents all the students in a class. The set M represents the students who take Music. The set D represents the students who take Drama. If 24 students take Music, calculate

- (i) the number of students who take BOTH Music and Drama (4 marks)
- (ii) the number of students who take Drama ONLY. (4 marks)
- (b) A straight line passes through the point $P(-3, 5)$ and has a gradient of $\frac{2}{3}$.
- (i) Write down the equation of this line in the form $y = mx + c$. (5 marks)
- (ii) Show that this line is parallel to the line $2x - 3y = 0$. (2 marks)

Total 11 marks

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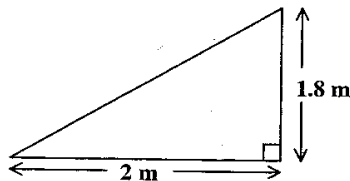
4. The figures shown below, **not drawn to scale**, represent the cross sections of two circular pizzas. Both pizzas are equally thick and contain the same toppings.



- (a) Is a medium pizza twice as large as a small pizza?
Use calculations to support your answer. (5 marks)
- (b) A medium pizza is cut into 3 equal parts, and each part is sold for \$15.95. A small pizza is sold for \$12.95.
Which is the better buy?
Use calculations to support your answer. (5 marks)

Total 10 marks

5. (a) On graph paper, draw the x -axis and the y -axis. Using a scale of 1 cm to represent 1 unit on both axes, draw the triangle DEF with vertices $D(1, 1)$, $E(3, 1)$ and $F(1, 4)$. (3 marks)
- (b) (i) Draw the image of $\triangle DEF$ under reflection in the line $x = 4$. Name the image $\triangle D'E'F'$.
- (ii) Draw the image of $\triangle D'E'F'$ under the translation $\begin{bmatrix} 0 \\ -5 \end{bmatrix}$. Name the image $\triangle D''E''F''$.
- (iii) Name the type of transformation that maps $\triangle DEF$ onto $\triangle D''E''F''$. (5 marks)
- (c) A vertical stick of height 1.8 m casts a shadow of length 2 m on the horizontal as shown in the diagram below, **not drawn to scale**.

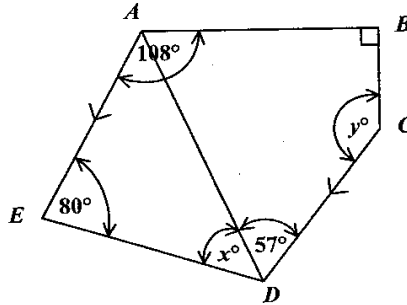


Calculate, to the NEAREST degree, the angle of elevation of the sun. (4 marks)

Total 12 marks

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6. (a) In the diagram shown below, $ABCDE$ is a pentagon. $\angle BAE = 108^\circ$, $\angle ABC = 90^\circ$, $\angle AED = 80^\circ$, $\angle ADC = 57^\circ$ and AE is parallel to CD .



Calculate the size of the angle marked

(i) x°

(ii) y°

(4 marks)

Show all steps in your calculations and give reasons for your answers.

- (b) The functions f and g are defined by

$$f(x) = \frac{1}{2}x + 5, \quad g(x) = x^2.$$

Evaluate

(i) $g(3) + g(-3)$

(ii) $f^{-1}(6)$

(iii) $fg(2)$

(8 marks)

Total 12 marks

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7. The table below gives the distribution of heights of 400 female applicants for the Police Service.

Height (cm)	Number of Applicants	Cumulative Frequency
151 - 155	10	10
156 - 160	55	65
161 - 165	105	170
166 - 170	110	280
171 - 175	80	360
176 - 180	30	390
181 - 185	10	400

- (a) Using a horizontal scale of 2 cm to represent a height of 5 cm and a vertical scale of 2 cm to represent 50 applicants, draw a cumulative frequency curve of the heights.

Start your horizontal scale at 150 cm. (5 marks)

- (b) Use your graph to estimate
- (i) the number of applicants whose heights are less than 170 cm. (1 mark)
 - (ii) the median height of applicants. (2 marks)
 - (iii) the height that 25% of the applicants are less than (2 marks)
 - (iv) the probability that an applicant selected at random has a height that is no more than 162 cm. (2 marks)

Credit will be given for drawing appropriate lines on your graph to show how the estimates were obtained.

Total 12 marks

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8. (a) Study the number pattern in the table below and complete lines (i), (ii) and (iii) in your answer booklet.

	2^3	$(0 \times 3^2) + (3 \times 2) + 2$	8
	3^3	$(1 \times 4^2) + (3 \times 3) + 2$	27
	4^3	$(2 \times 5^2) + (3 \times 4) + 2$	64
	5^3	$(3 \times 6^2) + (3 \times 5) + 2$	125
(i)	6^3		
(ii)	10^3		
(iii)	n^3	$(n-2) \times ()^2 + (3 \times) + 2$	n^3

(7 marks)

- (b) Show that

$$(a-b)^2(a+b) + ab(a+b) = a^3 + b^3.$$

(3 marks)

Total 10 marks

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SECTION II

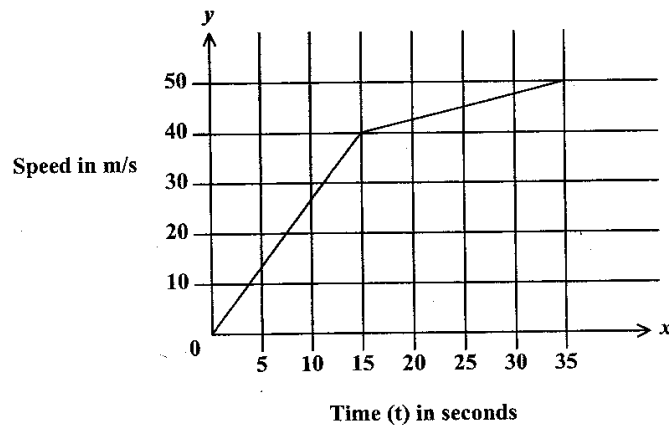
Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) Write $5x^2 + 2x - 7$ in the form $a(x+b)^2 + c$, where a , b , and c are real numbers. (4 marks)
- (b) Hence, or otherwise, determine
- (i) the minimum value of the function $y = 5x^2 + 2x - 7$
- (ii) the value of x at which the minimum occurs (3 marks)
- (c) Find the values of x for which $5x^2 + 2x - 7 = 0$. (3 marks)
- (d) Sketch the graph of $y = 5x^2 + 2x - 7$, clearly showing
- (i) the coordinates of the minimum point
- (ii) the value of the y -intercept
- (iii) the points where the graph cuts the x -axis. (5 marks)

Total 15 marks

10. (a) The speed-time graph below shows the movement of a cyclist.

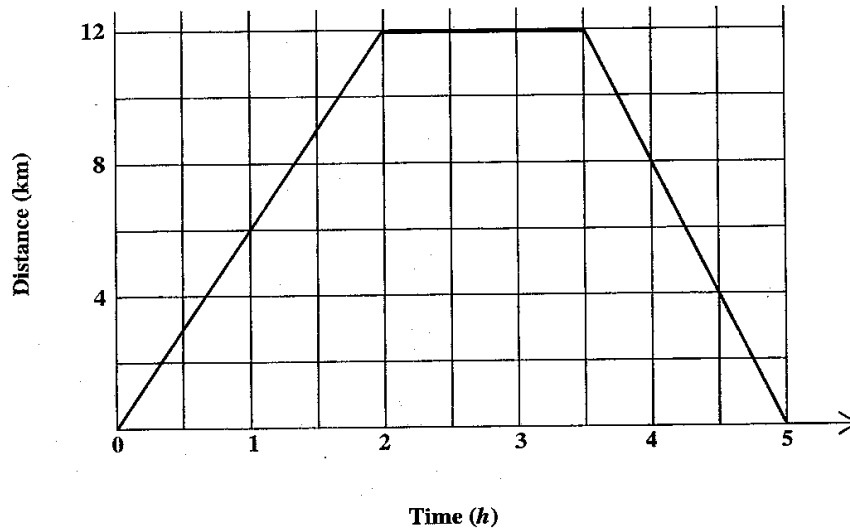


Using the graph, calculate

- (i) the acceleration of the cyclist during the first 15 seconds
- (ii) the distance traveled by the cyclist between the period $t = 15$ and $t = 35$ seconds. (6 marks)

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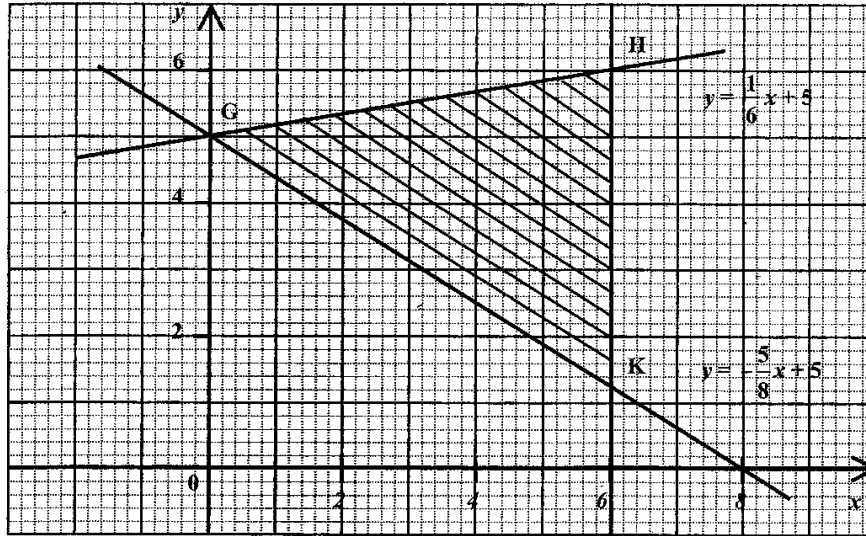
- (b) The graph below represents the 5-hour journey of an athlete.



- (i) What was the average speed during the first 2 hours?
- (ii) What did the athlete do between 2 and 3 hours after the start of the journey?
- (iii) What was the average speed on the return journey? **(5 marks)**

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- (c) The diagram below shows a triangular region bounded by the lines $y = \frac{1}{6}x + 5$, $y = -\frac{5}{8}x + 5$ and the line HK .



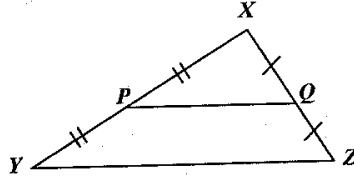
- (i) Write the equation of the line HK . (1 mark)
- (ii) Write the set of **three inequalities** which define the shaded region GHK . (3 marks)

Total 15 marks

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GEOMETRY AND TRIGONOMETRY

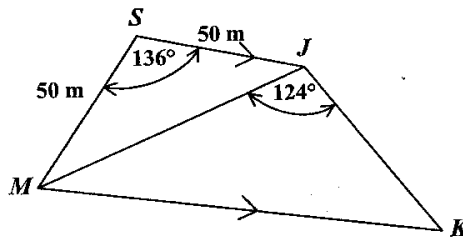
11. (a)



In the diagram above, **not drawn to scale**, P and Q are midpoints of the sides XY and XZ of triangle XYZ . Given that $XP = 7.5$ cm, $XQ = 4.5$ cm and the area of triangle $XPQ = 13.5$ cm², calculate

- (i) the size of angle PXQ , expressing your answer correct to the nearest degree.
- (ii) the area of triangle YXZ . (6 marks)

(b)



The figure $SJKM$ above, **not drawn to scale**, is a trapezium with SJ parallel to MK , angle $MJK = 124^\circ$, angle $MSJ = 136^\circ$, and $SM = SJ = 50$ metres.

- (i) Calculate the size of
 - a) angle SJM
 - b) angle JKM . (3 marks)
- (ii) Calculate, expressing your answer correct to ONE decimal place, the length of
 - a) MJ
 - b) JK . (6 marks)

Total 15 marks

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12. In this question, assume the earth to be a sphere of radius 6 400 km and use $\pi = 3.14$.

The latitudes and longitudes of Antigua and of Belize are given in the table below.

Country	Latitude	Longitude
Antigua	17°N	62°W
Belize	17°N	88°W

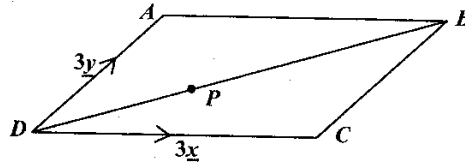
- (a) Draw a sketch of the earth showing the location of Antigua and of Belize, their associated circles of latitude and longitude, the equator, and the Greenwich Meridian. (6 marks)
- (b) Calculate the shortest distance between Antigua and Belize measured along their common circle of latitude. (5 marks)
- (c) A town, Bahia Blanka, situated in South America, lies on a meridian 62°W and has a latitude of 38°S. Calculate the shortest distance between Antigua and Bahia Blanka measured along the common circle of longitude. (4 marks)

Total 15 marks

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VECTORS AND MATRICES

13.



In the figure above, **not drawn to scale**, $ABCD$ is a parallelogram such that $\vec{DC} = 3\underline{x}$ and $\vec{DA} = 3\underline{y}$. The point P is on DB such that $DP : PB = 1:2$.

- (a) Express in terms of \underline{x} and \underline{y} :
- \vec{AB}
 - \vec{BD}
 - \vec{DP} (5 marks)
- (b) Show that $\vec{AP} = \underline{x} - 2\underline{y}$. (2 marks)
- (c) Given that E is the mid-point of DC , prove that A , P and E are collinear. (4 marks)
- (d) Given that $x = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ and $y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, use a vector method to prove that triangle AED is isosceles. (4 marks)

Total 15 marks

14. (a) Given that $M = \begin{bmatrix} 2 & 5 \\ 7 & 15 \end{bmatrix}$.

- (i) Show that M is a non-singular matrix.
- (ii) Write down the inverse of M .
- (iii) Write down the 2×2 matrix which is equal to the product $M \times M^{-1}$.
- (iv) Pre-multiply both sides of the following matrix equation by M^{-1} .

$$\begin{bmatrix} 2 & 5 \\ 7 & 15 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 17 \end{bmatrix}$$

Hence solve for x and y .

(7 marks)

- (b) (i) Write down the 2×2 matrix, R , which represents a reflection in the y -axis.
- (ii) Write down the 2×2 matrix, N , which represents a clockwise rotation of 180° about the origin.
- (iii) Write down the 2×1 matrix, T which represents a translation of -3 units parallel to the x -axis and 5 units parallel to the y -axis.
- (iv) The point $P(6, 11)$ undergoes the following combined transformations such that

$RN(P)$ maps P onto P'
 $NT(P)$ maps P onto P''

Determine the coordinates of P' and P'' .

(8 marks)

Total 15 marks

END OF TEST