TEST CODE **01234020**

FORM TP 2007105

MAY/JUNE 2007

CARIBBEAN EXAMINATIONS COUNCIL

SECONDARY EDUCATION CERTIFICATE EXAMINATION

MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

24 MAY 2007 (a.m.)

INSTRUCTIONS TO CANDIDATES

1. Answer ALL questions in Section I, and ANY TWO in Section II.
2. Write your answers in the booklet provided.
3. All working must be shown clearly.
4. A list of formulae is provided on page 2 of this booklet.

Examination Materials

Electronic calculator (non-programmable)
Geometry set
Mathematical tables (provided)
Graph paper (provided)

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LIST OF FORMULAE

Volume of a prism — $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of cylinder — $V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.

Volume of a right pyramid — $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference — $C = 2\pi r$ where r is the radius of the circle.

Area of a circle — $A = \pi r^2$ where r is the radius of the circle.

Area of trapezium — $A = \frac{1}{2}(a + b)\,h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

Roots of quadratic equations — If $ax^2 + bx + c = 0$,

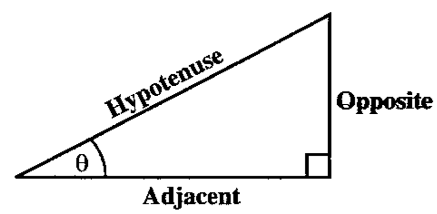
then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios

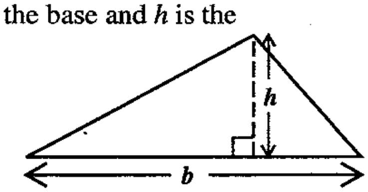
$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$

$\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

$\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$



Area of triangle — Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height



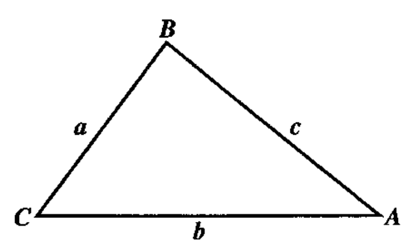
Area of $\Delta ABC = \frac{1}{2}ab\sin C$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \dfrac{a+b+c}{2}$

Sine rule — $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule — $a^2 = b^2 + c^2 - 2bc\cos A$



SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, determine the exact value of $(3.7)^2 - (6.24 \div 1.3)$. **(3 marks)**

(b) A total of 1 200 students attend Top View High School.

The ratio of teachers to students is 1:30.

(i) How many teachers are there at the school? **(2 marks)**

Two-fifths of the students own personal computers.

(ii) How many students do NOT own personal computers? **(2 marks)**

Thirty percent of the students who own personal computers also own play stations.

(iii) What **fraction** of the students in the school own play stations?

Express your answer in its **lowest** terms. **(4 marks)**

Total 11 marks

2. (a) Given that $a * b = ab - \frac{b}{a}$.

Evaluate

(i) $4 * 8$

(ii) $2 * (4 * 8)$ **(4 marks)**

(b) Simplify, expressing your answer in its simplest form

$\frac{5p}{3q} \div \frac{4p^2}{q}$ **(2 marks)**

(c) A stadium has two sections, A and B.
Tickets for Section A cost $\$a$ each.
Tickets for Section B cost $\$b$ each.

Johanna paid $105 for 5 Section A tickets and 3 Section B tickets.

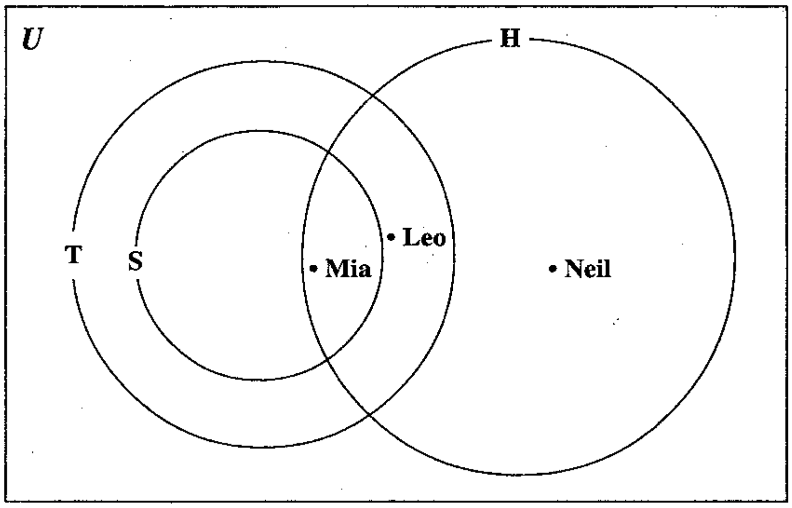
Raiyah paid $63 for 4 Section A tickets and 1 Section B ticket.

(i) Write two equations in a and b to represent the information above.

(ii) Calculate the values of a and b. **(5 marks)**

Total 11 marks

3. (a) The Venn Diagram below represents information on the type of games played by members of a youth club. All members of the club play **at least one** game.



S represents the set of members who play squash.
T represents the set of members who play tennis.
H represents the set of members who play hockey.

Leo, Mia and Neil are three members of the youth club.

(i) State what game(s) is/are played by

a) Leo

b) Mia

c) Neil

(ii) Describe in words the members of the set $H' \cap S$. **(5 marks)**

(b) (i) Using a pencil, a ruler and a pair of compasses only.

a) Construct a triangle PQR in which $QR = 8.5$ cm, $PQ = 6$ cm and $PR = 7.5$ cm.

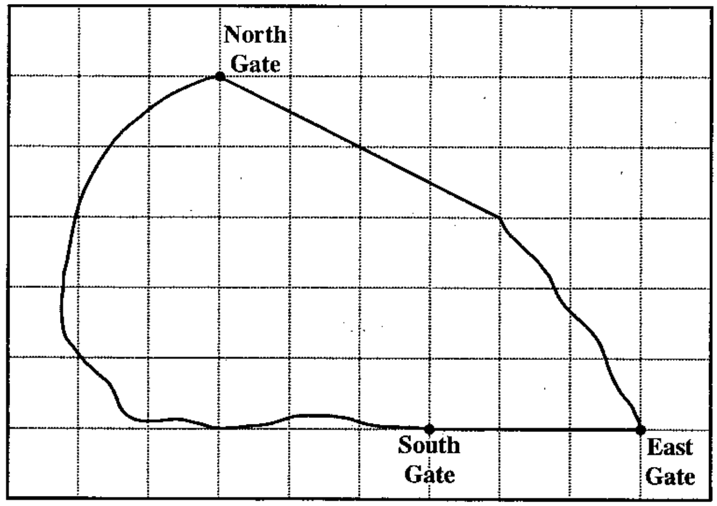
b) Construct a line PT such that PT is perpendicular to QR and meets QR at T.

(ii) a) Measure and state the size of angle PQR.

b) Measure and state the length of PT. **(7 marks)**

Total 12 marks

4. (a) The diagram below shows a map of a golf course drawn on a grid of 1 cm squares. The scale of the map is 1:4000.



Using the map of the golf course, find

(i) the distance, to the nearest m, from South Gate to East Gate

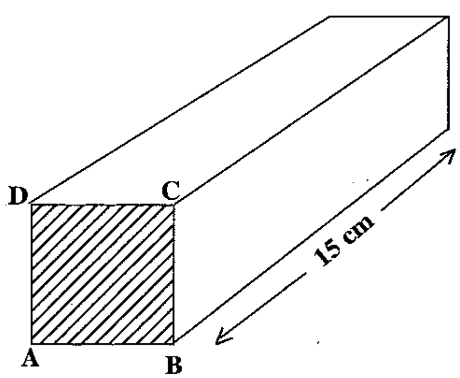
(ii) the distance, to the nearest m, from North Gate to South Gate

(iii) the area on the ground represented by 1 cm^2 on the map

(iv) the actual area of the golf course, giving the answer in square metres.

(6 marks)

(b) The diagram below, **not drawn to scale**, shows a prism of volume 960 cm^3. The cross-section ABCD is a square. The length of the prism is 15 cm.



Calculate

(i) the length of the edge AB, in cm

(ii) the total surface area of the prism, in cm^2.

(5 marks)

Total 11 marks

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5. Two variables x and y are related such that 'y varies inversely as the square of x'.

(a) Write an equation in x, y and k to describe the inverse variation, where k is the constant of variation. **(2 marks)**

(b)

x	3	1.8	f
y	2	r	8

Using the information in the table above, calculate the value of

(i) k, the constant of variation

(ii) r

(iii) f. **(6 marks)**

(c) Determine the equation of the line which is parallel to the line $y = 2x + 3$ and passes through the coordinate (4,7). **(4 marks)**

Total 12 marks

6. (a) **An answer sheet is provided for this question.**

$L'M'N'$ is the image of LMN under an enlargement.

(i) Write on your answer sheet

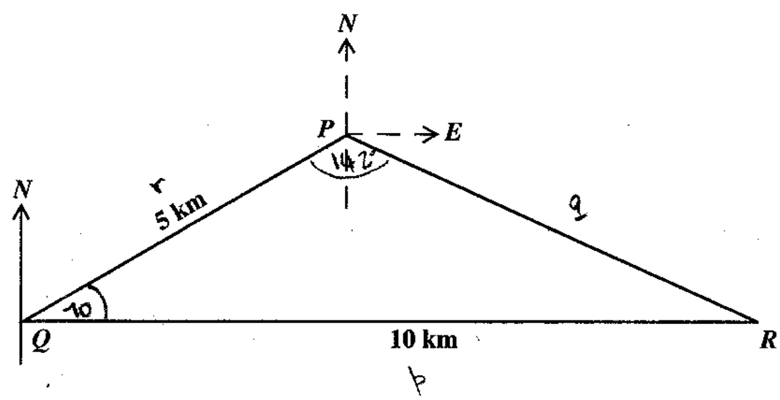
a) the scale factor for the enlargement

b) the coordinates of the centre of the enlargement.

$L''M''N''$ is the image of LMN under a reflection in the line $y = -x$.

(ii) Draw and label the triangle $L''M''N''$ on your answer sheet. **(5 marks)**

(b)



Three towns, P, Q and R are such that the bearing of P from Q is 070°. R is 10 km due east of Q and $PQ = 5$ km.

(i) Calculate, correct to one decimal place, the distance PR.

(ii) Given that $\angle QPR = 142°$, state the bearing of R from P. **(6 marks)**

Total 11 marks

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7. A class of 32 students participated in running a 400 m race in preparation for their sports day. The time, in seconds, taken by each student is recorded below.

83	51	56	58	62	65	61	64
72	71	54	62	81	80	78	77
71	55	70	54	82	59	71	62
83	63	65	72	78	73	68	75

(a) Copy and complete the frequency table to represent this data.

Time in seconds	**Frequency**
50 – 54	3
55 – 59	4
60 – 64	6
65 – 69	
70 – 74	
75 – 79	
80 – 84	

(2 marks)

(b) Using the raw scores, determine the range for the data. **(2 marks)**

(c) Using a scale of 2 cm to represent 5 seconds on the horizontal axis and a scale of 1 cm to represent 1 student on the vertical axis, draw a frequency polygon to represent the data.

NOTE: An empty interval must be shown at each end of the distribution and the polygon closed. **(6 marks)**

(d) To qualify for the finals, a student must complete the race in less than 60 seconds. What is the probability that a student from this class will qualify for the finals?

(2 marks)

Total 12 marks

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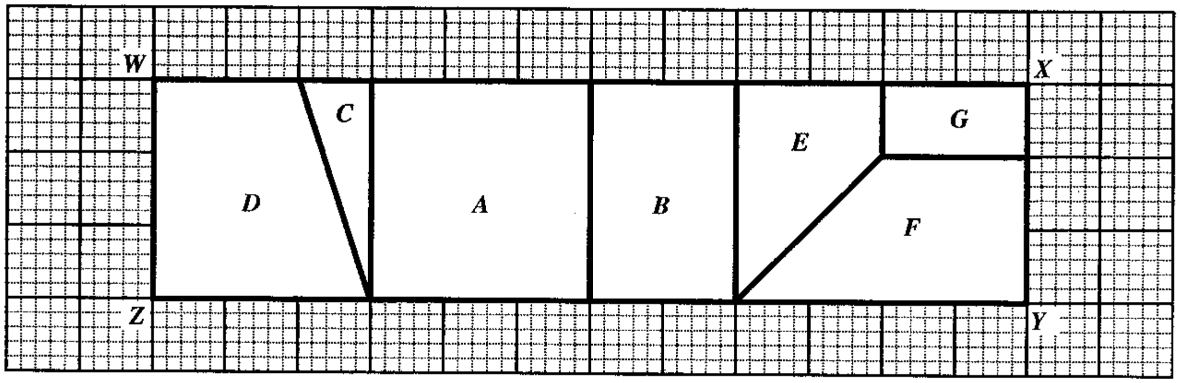
8. Rectangle *WXYZ* below represents one whole unit which has been divided into seven smaller parts. These parts are labelled *A*, *B*, *C*, *D*, *E*, *F* and *G*.



(a) Copy and complete the following table, stating what fraction of the rectangle each part represents.

Part	Fraction
A	
B	
C	$\frac{1}{24}$
D	
E	
F	
G	$\frac{1}{18}$

(5 marks)

(b) Write the parts in order of the size of their perimeters. **(2 marks)**

(c) The area of G is 2 square units. E, F and G are rearranged to form a trapezium.

(i) What is the area of the trapezium in square units?

(ii) Sketch the trapezium clearly showing the outline of each of the three parts.

(3 marks)

Total 10 marks

SECTION II

Answer TWO questions in this section.

RELATIONS, FUNCTIONS AND GRAPHS

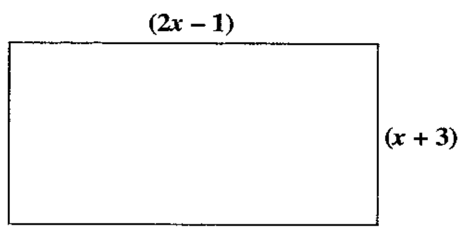
9. (a) Given that $g(x) = \dfrac{2x + 1}{5}$ and $f(x) = x + 4$.

(i) Calculate the value of $g\,(-2)$.

(ii) Write an expression for $gf(x)$ in its simplest form.

(iii) Find the inverse function $g^{-1}(x)$. **(7 marks)**

(b) The length of the rectangle below is $(2x - 1)$ cm and its width is $(x + 3)$ cm.



(i) Write an expression in the form $ax^2 + bx + c$ for the area of the rectangle.

(ii) Given that the area of the rectangle is 294 cm^2, determine the value of x.

(iii) Hence, state the dimensions of the rectangle, in centimetres. **(8 marks)**

Total 15 marks

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10. A company manufactures gold and silver stars to be used as party decorations. The stars are placed in packets so that each packet contains x gold stars and y silver stars.

The conditions for packaging are given in the table below.

Condition	Inequality
(1) Each packet must have at least 20 gold stars	$x \geq 20$
(2) Each packet must have at least 15 silver stars	
(3) The total number of stars in each packet must not be more than 60.	
(4)	$x < 2y$

(a) Write down the inequalities to represent conditions (2) and (3). **(2 marks)**

(b) Describe, in words, the condition represented by the inequality $x < 2y$. **(2 marks)**

(c) Using a scale of 2 cm to represent 10 units on both axes, draw the graphs of ALL FOUR inequalities represented in the table above. **(7 marks)**

(d) Three packets of stars were selected for inspection. Their contents are shown below.

Packet	No. of gold stars (x)	No. of silver stars (y)
A	25	20
B	35	15
C	30	25

Plot the points A, B and C on your graph. Hence determine which of the three packets satisfy ALL the conditions. **(4 marks)**

Total 15 marks

GEOMETRY AND TRIGONOMETRY

11. (a) Given that $\sin\theta = \frac{\sqrt{3}}{2}$.

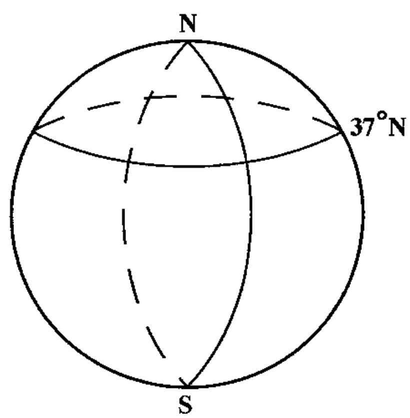
(i) Express in fractional or surd form

a) $\cos\theta$

b) $\tan\theta$.

(ii) Hence, determine the exact value of $\frac{\sin\theta}{\tan\theta}$. **(7 marks)**

(b) For this question take $\pi = 3.14$ and $R = 6\ 370$ km, where R is the radius of the earth. The diagram below, **not drawn to scale**, shows a sketch of the earth with the North and South poles labelled N and S respectively. The circle of latitude 37°N is shown.

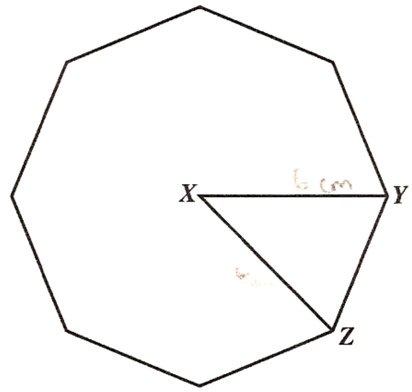


(i) Calculate, correct to the **nearest** kilometre, the length of the circle of latitude 37° N.

(ii) Two towns, A and B, have co-ordinates (37° N, 50° W) and (37° N, x° E) respectively. The distance from A to B measured along their common circle of latitude is 5 390 km, calculate the value of x. **(8 marks)**

Total 15 marks

12. (a) The figure below, **not drawn to scale**, is a regular octagon with centre X, and $XY = 6$ cm.



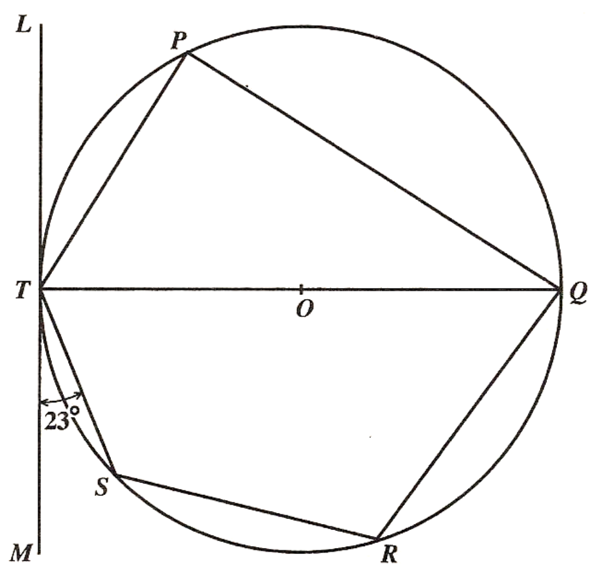
Calculate

(i) the size of angle YXZ

(ii) the area of the triangle YXZ, expressing your answer correct to one decimal place

(iii) the area of the octagon. **(6 marks)**

(b) In the diagram below, **not drawn to scale**, LM is a tangent to the circle at the point, T. O is the centre of the circle and angle $\angle MTS = 23°$.



Calculate the size of each of the following angles, giving reasons for your answer

a) angle TPQ

b) angle MTQ

c) angle TQS

d) angle SRQ. **(9 marks)**

Total 15 marks

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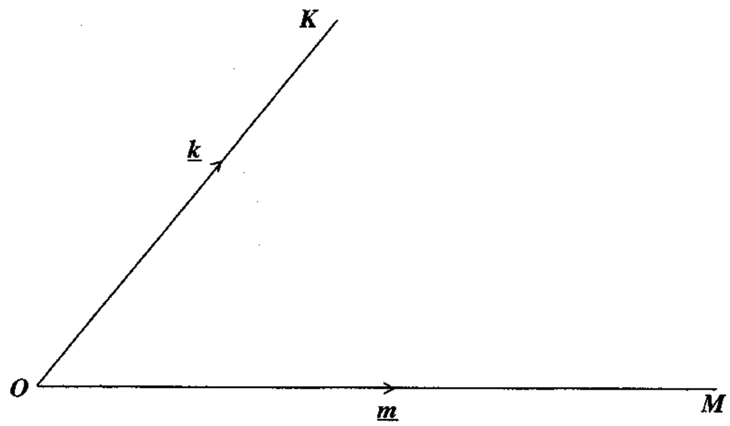
VECTORS AND MATRICES

13.



OK and OM are position vectors such that $\overrightarrow{OK} = \underline{k}$ and $\overrightarrow{OM} = \underline{m}$.

(a) Sketch the diagram above. Show the approximate positions of points R and S such that

R is the mid-point of OK

S is a point on OM such that $\overrightarrow{OS} = \frac{1}{3}\overrightarrow{OM}$. **(2 marks)**

(b) Write down, in terms of $\underline{k}$ and $\underline{m}$ the vectors

(i) $\overrightarrow{MK}$

(ii) $\overrightarrow{RM}$

(iii) $\overrightarrow{KS}$

(iv) $\overrightarrow{RS}$. **(8 marks)**

(c) L is the mid-point of RM. **Using a vector method**, prove that RS is parallel to KL. **(5 marks)**

Total 15 marks

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14. (a) A, B and C are three 2 x 2 matrices such that $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $B = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$, *and* $C = \begin{pmatrix} 14 & 0 \\ -9 & 5 \end{pmatrix}$.

Find

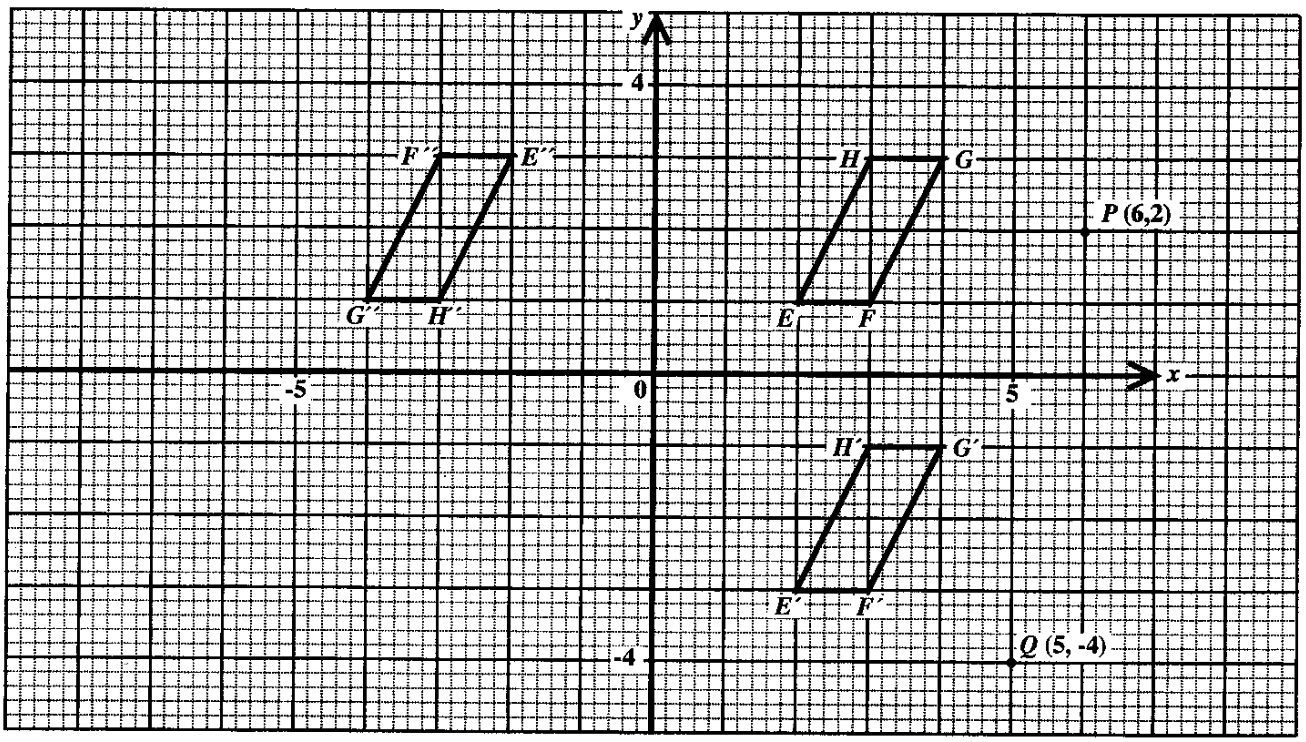
(i) $3A$

(ii) B^{-1}

(iii) $3A + B^{-1}$

(iv) the value of a, b, c and d given that $3A + B^{-1} = C$. **(7 marks)**

(b) The diagram below shows a parallelogram *EFGH* and its images after undergoing two successive transformations.



(i) Describe in words, the geometric transformations

a) J which maps $EFGH$ onto $E'F'G'H'$

b) K which maps $E'F'G'H'$ onto $E''F''G''H''$.

(ii) Write the matrix which represents the transformation described above as

a) J

b) K

(iii) The point P (6, 2) is mapped onto P' by the transformation J. State the co-ordinates of P′.

(iv) The point Q (5, –4) is mapped onto Q' by the transformation K. State the co-ordinates of Q′. **(8 marks)**

Total 15 marks

END OF TEST