INSTRUCTIONS TO CANDIDATES

1. This paper consists of TWO sections.
2. There are EIGHT questions in Section I and THREE questions in Section II.
3. Answer ALL questions in Section I, and any TWO questions from Section II.
4. Write your answers in the booklet provided.
5. All working must be clearly shown.
6. A list of formulae is provided on page 2 of this booklet.

Required Examination Materials

Electronic calculator
Geometry set
Graph paper (provided)
LIST OF FORMULAE

Volume of a prism \[ V = Ah \] where \( A \) is the area of a cross-section and \( h \) is the perpendicular length.

Volume of cylinder \[ V = \pi r^2h \] where \( r \) is the radius of the base and \( h \) is the perpendicular height.

Volume of a right pyramid \[ V = \frac{1}{3} Ah \] where \( A \) is the area of the base and \( h \) is the perpendicular height.

Circumference \[ C = 2\pi r \] where \( r \) is the radius of the circle.

Arc length \[ S = \frac{\theta}{360} \times 2\pi r \] where \( \theta \) is the angle subtended by the arc, measured in degrees.

Area of a circle \[ A = \pi r^2 \] where \( r \) is the radius of the circle.

Area of a sector \[ A = \frac{\theta}{360} \times \pi r^2 \] where \( \theta \) is the angle of the sector, measured in degrees.

Area of trapezium \[ A = \frac{1}{2} (a + b) h \] where \( a \) and \( b \) are the lengths of the parallel sides and \( h \) is the perpendicular distance between the parallel sides.

Roots of quadratic equations If \( ax^2 + bx + c = 0 \),
then \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Trigonometric ratios
\[ \sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} \]
\[ \cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \]
\[ \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} \]

Area of triangle
Area of \( \Delta = \frac{1}{2} bh \) where \( b \) is the length of the base and \( h \) is the perpendicular height.

Area of \( \Delta ABC = \frac{1}{2} ab \sin C \)

Area of \( \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} \)
where \( s = \frac{a+b+c}{2} \)

Sine rule \[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

Cosine rule \[ a^2 = b^2 + c^2 - 2bc \cos A \]

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SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, determine the EXACT value of:

   (i) \[ \frac{2 \frac{1}{4} + 1 \frac{1}{8}}{4 \frac{1}{2}} \]
   expressing your answer as a fraction \hspace{1cm} (3 marks)

   (ii) \[ 3.96 \times 0.25 - \sqrt{0.0256} \]
   \hspace{1cm} (3 marks)

(b) The table below shows Pamela’s shopping bill. Some of the information was not included.

<table>
<thead>
<tr>
<th>Items</th>
<th>Quantity</th>
<th>Unit Price $</th>
<th>Total Cost $</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rice</td>
<td>6 ( \frac{1}{2} ) kg</td>
<td>2.40</td>
<td>( W )</td>
</tr>
<tr>
<td>Potatoes</td>
<td>4 bags</td>
<td>( X )</td>
<td>52.80</td>
</tr>
<tr>
<td>Milk</td>
<td>( Y ) cartons</td>
<td>2.35</td>
<td>14.10</td>
</tr>
<tr>
<td>Sub Total</td>
<td></td>
<td></td>
<td>82.50</td>
</tr>
<tr>
<td>( Z ) % VAT</td>
<td></td>
<td></td>
<td>9.90</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td></td>
<td>92.40</td>
</tr>
</tbody>
</table>

Calculate the values of \( W, X, Y \) and \( Z \). \hspace{1cm} (5 marks)

Total 11 marks
2. (a) Write as a single fraction in its lowest terms
\[
\frac{x-2}{3} + \frac{x+1}{4}
\] (3 marks)

(b) The binary operation \( \ast \) is defined by
\[
a \ast b = (a + b)^2 - 2ab.
\]
Calculate the value of \( 3 \ast 4 \). (2 marks)

(c) Factorise completely
(i) \( xy^3 + x^2y \) (2 marks)
(ii) \( 2mh - 2nh - 3mk + 3nk \) (2 marks)

(d) The table below shows corresponding values of the variables \( x \) and \( y \), where \( y \) varies directly as \( x \).

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>2</td>
<td>5</td>
<td>( b )</td>
</tr>
<tr>
<td>( y )</td>
<td>12</td>
<td>( a )</td>
<td>48</td>
</tr>
</tbody>
</table>

Calculate the values of \( a \) and \( b \). (3 marks)

Total 12 marks
3. (a) The Venn diagram below shows the number of students who study Music and Art in a class of 35 students.

\[ U = \{ \text{students in the class}\} \]
\[ M = \{ \text{students who study Music}\} \]
\[ A = \{ \text{students who study Art}\} \]

![Venn Diagram]

(i) How many students study neither Art nor Music? (1 mark)

(ii) Calculate the value of \( x \). (3 marks)

(iii) Hence, state the number of students who study Music only. (1 mark)

(b) (i) Using a ruler, pencil, a pair of compasses and a protractor, draw accurately a quadrilateral \( EFGH \) using the following measurements:

\[ EF = 8 \text{ cm} \quad <EFG = 125^\circ \quad FG = 4 \text{ cm} \quad <HEF = 70^\circ \quad EH = 7 \text{ cm} \]

(5 marks)

(ii) Measure and state in centimetres, the length of \( GH \). (1 mark)

Total 11 marks
4. (a) (i) Solve the inequality: $5 - 2x < 9$  

(ii) If $x$ is an integer, determine the SMALLEST value of $x$ that satisfies the inequality in (a) (i) above. 

(b) In this question, use $\pi = \frac{22}{7}$. 

(i) A piece of wire is bent to form a square of area 121 cm$^2$. 

Calculate: 

a) The length of each side of the square  

b) The perimeter of the square  

(ii) The same piece of wire is bent to form a circle. 

Calculate: 

a) The radius of the circle  

b) The area of the circle  

Total 10 marks
5. (a) The diagram below, not drawn to scale, shows $\triangle OMN$, and its image, $\triangle OM'N'$ under an enlargement with centre, $O$, and scale factor, $k$. Angle $ONM = 90^\circ$.

![Diagram of $\triangle OMN$ and $\triangle OM'N'$]

Using the dimensions shown on the diagram, calculate

(i) the value of $k$, the scale factor of the enlargement

(ii) the length of $OM$

(iii) the length of $OM'$.

(b) The diagram below, not drawn to scale, shows $\triangle PQR$, which represents the cross section of a roof. $QS$ is perpendicular to $PSR$.

$PQ = 12.6$ metres $\quad QR = 8.4$ metres $\quad \angle QPR = 15^\circ$

![Diagram of $\triangle PQR$]

Using the dimensions shown on the diagram, calculate, correct to 3 significant figures

(i) the length of $QS$

(ii) the measure of $\angle QOS$

(iii) the area of $\triangle PQR$.

Total 12 marks
6. (a) The functions $f$ and $g$ are defined by
\[ f(x) = 6x + 8 \quad ; \quad g(x) = \frac{x - 2}{3} \, . \]

(i) Calculate the value of $g\left(\frac{1}{2}\right)$. \hspace{1cm} \hspace{1cm} (2 marks)

(ii) Write an expression for $g\, f(x)$ in its simplest form. \hspace{1cm} (2 marks)

(iii) Find the inverse function $f^{-1}(x)$. \hspace{1cm} (2 marks)

(b) The diagram below shows the line segment which passes through the points $A$ and $B$.

![Graph with points A and B]

Determine

(i) the coordinates of $A$ and $B$ \hspace{1cm} (2 marks)

(ii) the gradient of the line segment $AB$ \hspace{1cm} (2 marks)

(iii) the equation of the line which passes through $A$ and $B$. \hspace{1cm} (2 marks)

Total 12 marks
7. The table below shows the distribution of the masses of 100 packages.

<table>
<thead>
<tr>
<th>Mass (kg)</th>
<th>No. of Packages</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 10</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>11 – 20</td>
<td>28</td>
<td>40</td>
</tr>
<tr>
<td>21 – 30</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>31 – 40</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>41 – 50</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

(a) Copy and complete the table to show the cumulative frequency for the distribution. 
    (2 marks)

(b) Using a scale of 2 cm to represent 10 kg on the x-axis and 1 cm to represent 10 packages on the y-axis, draw the cumulative frequency curve for the data. 
    (5 marks)

(c) Estimate from the graph

(i) the median mass of the packages 
    (2 marks)

(ii) the probability that a package, chosen at random, has a mass which is LESS than 35 kg. 
    (3 marks)

Total 12 marks
8. An answer sheet is provided for this question.

The figure below shows the first three diagrams in a sequence. Each diagram is made up of sticks joined at the ends by thumb tacks. The sticks are represented by lines and the thumb tacks by dots. In each diagram, there are \( t \) thumb tacks and \( s \) sticks.

<table>
<thead>
<tr>
<th>Diagram 1</th>
<th>Diagram 2</th>
<th>Diagram 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**On the answer sheet provided:**

(a) Draw the FOURTH diagram in the sequence.  

(b)  

(i) How many sticks are in the SIXTH diagram?  

(ii) How many thumb tacks are in the SEVENTH diagram?  

(c) Complete the table by inserting the missing values at the rows marked (i) and (ii).

<table>
<thead>
<tr>
<th>No. of Sticks ( s )</th>
<th>Rule Connecting ( t ) and ( s )</th>
<th>No. of Thumb Tacks ( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>( 1 + \left( \frac{3}{4} \times 4 \right) )</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>( 1 + \left( \frac{3}{4} \times 8 \right) )</td>
<td>7</td>
</tr>
<tr>
<td>12</td>
<td>( 1 + \left( \frac{3}{4} \times 12 \right) )</td>
<td>10</td>
</tr>
</tbody>
</table>

(i)  

(ii)  

(d) Write the rule, in terms of \( s \) and \( t \), to show how \( t \) is related to \( s \).  

**Total 10 marks**
SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9.  (a) Solve the pair of simultaneous equations

\[ y = x^2 - x + 3 \]
\[ y = 6 - 3x \]  (5 marks)

(b) (i) Express the function \( f(x) = 4x^2 - 8x - 2 \) in the form \( a(x + h)^2 + k \), where \( a \), \( h \) and \( k \) are constants.  (2 marks)

(ii) the minimum value of \( f(x) \)  (1 mark)

(iii) the value of \( x \) for which \( f(x) \) is a minimum.  (1 mark)

(c) The speed-time graph below, not drawn to scale, shows the three-stage journey of a racing car over a period of 60 seconds.

During the FIRST stage of the journey, the car increased its speed from 0 m/s to 12 m/s in \( x \) seconds accelerating at 0.6 m/s².

(i) Calculate the value of \( x \).  (2 marks)

(ii) What is the gradient of the graph during the SECOND stage? Explain, in one sentence, what the car is doing during this stage.  (2 marks)

(iii) Calculate the distance travelled by the car on the THIRD stage of the journey.  (2 marks)

Total 15 marks

GO ON TO THE NEXT PAGE
MEASUREMENT, GEOMETRY AND TRIGONOMETRY

10. (a) In the diagram below, not drawn to scale, $W, X, Y$ and $Z$ are points on the circumference of a circle, centre $O$. $TYV$ is a tangent to the circle at $Y$. $\angle XWZ = 64^\circ$ and $\angle ZYV = 23^\circ$.

\[
\begin{array}{c}
\text{W} \\
\text{64} \\
\text{O} \\
\text{X} \\
\text{Y} \\
\text{Z} \\
\text{T} \\
\text{V}
\end{array}
\]

Calculate, giving reasons for your answer, the measure of angle

(i) $\angle XYZ$ 

(ii) $\angle YXZ$ 

(iii) $\angle OXZ$. 

(b) The diagram below, not drawn to scale, shows the route of an aeroplane flying from Portcity ($P$) to Queenstown ($Q$) and then to Riversdale ($R$). The bearing of $Q$ from $P$ is $132^\circ$ and the angle $PQO$ is $56^\circ$.

\[
\begin{array}{c}
\text{N} \\
\text{P} \\
132^\circ \\
\text{Q} \\
\text{R}
\end{array}
\]

(i) Calculate the value of $x$, as shown in the diagram.

(ii) The distance from Portcity ($P$) to Queenstown ($Q$) is 220 kilometres and the distance from Queenstown to Riversdale ($R$) is 360 kilometres. Calculate the distance $RP$.

(iii) Determine the bearing of $R$ from $P$.

Total 15 marks

GO ON TO THE NEXT PAGE.
VECTORS AND MATRICES

11. (a) Determine the inverse of the matrix \[
\begin{pmatrix}
3 & 5 \\
2 & 4
\end{pmatrix}
\] (2 marks)

(b) The transformation, \( M = \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} \), maps the points \( R \) and \( T \) onto \( R' \) and \( T' \) such that:

\[
R (7, 2) \longrightarrow R' (2, -7) \text{ and } T (-5, 4) \longrightarrow T' (4, 5)
\]

(i) Determine the values of \( a \) and \( b \). (2 marks)

(ii) Describe fully the transformation, \( M \). (3 marks)

(c) \( WXYV \) is a parallelogram in which \( \overrightarrow{VY} = a \) and \( \overrightarrow{VW} = b \).

\( S \) is a point on \( WY \) such that \( WS : SY = 1 : 2 \).

\[\begin{array}{cccc}
W & & & X \\
& b & S' &  \\
V & & & Y \\
& a &  \\
\end{array}\]

(i) Write in terms of \( a \) and \( b \), an expression for:

a) \( \overrightarrow{WY} \)

b) \( \overrightarrow{WS} \)

c) \( \overrightarrow{SX} \) (5 marks)

(ii) \( R \) is the mid-point of \( VW \). Prove that \( R, S \) and \( X \) are collinear. (3 marks)

Total 15 marks

END OF TEST