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FORM TP 2014037

MAY/JUNE 2014

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

**CARIBBEAN SECONDARY EDUCATION CERTIFICATE®
EXAMINATION**

ADDITIONAL MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

06 MAY 2014 (p.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of FOUR sections. Answer ALL questions in Section 1, Section 2 and Section 3.
2. Answer ONE question in Section 4.
3. Write your solutions with full working in the booklet provided.
4. A list of formulae is provided on page 2 of this booklet.

Required Examination Materials

Electronic Calculator (non programmable)
Geometry Set
Mathematical Tables (provided)
Graph Paper (provided)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

LIST OF FORMULAE

Arithmetic Series $T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric Series $T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_\infty = \frac{a}{1-r},\ -1 < r < 1 \text{ or } |r| < 1$

Circle $x^2 + y^2 + 2fx + 2gy + c = 0 \qquad (x+f)^2 + (y+g)^2 = r^2$

Vectors $\hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \cos\theta = \frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{a}| \times |\mathbf{b}|} \qquad |\mathbf{v}| = \sqrt{(x^2 + y^2)}$ where $\mathbf{v} = x\mathbf{i} + y\mathbf{j}$

Trigonometry

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Differentiation

$$\frac{\mathrm{d}}{\mathrm{d}x}(ax+b)^n = an(ax+b)^{n-1}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$$

Statistics

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}, \qquad S^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n} = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\overline{x})^2$$

Probability $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Kinematics $v = u + at \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2$

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SECTION 1

Answer BOTH questions.

ALL working must be clearly shown.

1. (a) (i) The function f is defined by $f : x \rightarrow 1 - x^2, x \in \mathbb{R}$.

Show that f is NOT one-to-one. **(1 mark)**

(ii) The function g is defined by $g : x \rightarrow \frac{1}{2}x - 3, x \in \mathbb{R}$.

a) Find $fg(x)$, and clearly state its domain. **(2 marks)**

b) Determine the inverse, g^{-1}, of g and sketch on the same pair of axes, the graphs of g and g^{-1}. **(3 marks)**

(b) When the expression $2x^3 + ax^2 - 5x - 2$ is divided by $2x - 1$, the remainder is -3.5.

Determine the value of the constant a. **(3 marks)**

(c) The length of a regtangular kitchen is y m and the width is x m. If the length of the kitchen is half the square of its width and its perimeter is 48 m, find the values of x and y (the dimensions of the kitchen). **(5 marks)**

Total 14 marks

2. (a) Given that $f(x) = -2x^2 - 12x - 9$.

(i) Express $f(x)$ in the form $k + a(x + h)^2$, where a, h and k are integers to be determined. **(3 marks)**

(ii) State the maximum value of $f(x)$. **(1 mark)**

(iii) Determine the value of x for which $f(x)$ is a maximum. **(1 mark)**

(b) Find the set of values of x for which $3 + 5x - 2x^2 \leq 0$. **(4 marks)**

(c) A series is given by $0.2 + 0.02 + 0.002 + 0.0002 + \ldots$

(i) Show that this series is geometric. **(3 marks)**

(ii) Find the sum to infinity of this series, giving your answer as an exact fraction. **(2 marks)**

Total 14 marks

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SECTION 2

Answer BOTH questions.

ALL working must be clearly shown.

3. (a) (i) Determine the value of k such that the lines $x + 3y = 6$ and $kx + 2y = 12$ are perpendicular to each other. **(3 marks)**

(ii) A circle of radius 5 cm has as its centre the point of intersection of the two perpendicular lines in (i). Determine the equation for this circle. **(3 marks)**

(b) RST is a triangle in the coordinate plane. Position vectors R, S, and T relative to an origin, O, are $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ respectively.

(i) Show that $T\hat{R}S = 90°$. **(4 marks)**

(ii) Determine the length of the hypotenuse. **(2 marks)**

[Hint: A rough drawing of RST might help].

Total 12 marks

4. (a) Figure 1 shows the sector OAB of a circle with centre O, radius 9 cm and angle 0.7 radians.

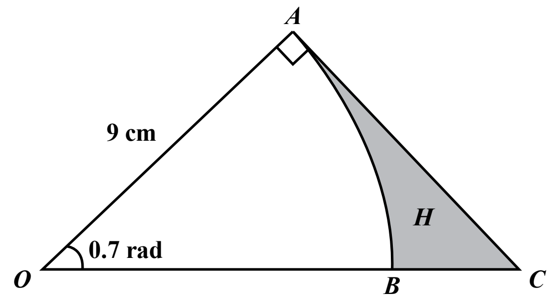

Figure 1.

(i) Find the area of the sector OAB. **(2 marks)**

(ii) Hence, find the area of the shaded region, H. **(4 marks)**

(b) Given that $\sin \frac{\pi}{6} = \frac{1}{2}$ and $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$, show that

$$\cos\left(x + \frac{\pi}{6}\right) = \frac{1}{2}\ (\sqrt{3}\ \cos x - \sin x),$$ where x is acute. **(2 marks)**

(c) Prove the identity $\left(\frac{\tan\theta\ \sin\theta}{1 - \cos\theta}\right) \equiv 1 + \frac{1}{\cos\theta}$. **(4 marks)**

Total 12 marks

SECTION 3

Answer BOTH questions.

ALL working must be clearly shown.

5. (a) The equation of a curve is $y = 3 + 4x - x^2$. The point P (3, 6) lies on the curve.

Find the equation of the tangent to the curve at P, giving your answer in the form

$ax + by + c = 0$, where $a, b, c, \in \mathbb{Z}$. **(4 marks)**

(b) Given that $f(x) = 2x^3 - 9x^2 - 24x + 7$.

(i) Find ALL the stationary points of $f(x)$. **(5 marks)**

(ii) Determine the nature of EACH of the stationary points of $f(x)$. **(5 marks)**

Total 14 marks

6. (a) Evaluate $\int_2^4 x\,(x^2 - 2)\,dx$. **(4 marks)**

(b) Evaluate $\int_0^{\frac{\pi}{3}} (4\cos x + 2\sin x)\,dx$, leaving your answer in surd form. **(4 marks)**

(c) A curve passes through the point P (2, –5) and is such that $\frac{dy}{dx} = 6x^2 - 1$.

(i) Determine the equation of the curve. **(3 marks)**

(ii) Find the area of the finite region bounded by the curve, the x-axis, the line $x = 3$ and the line $x = 4$. **(3 marks)**

Total 14 marks

SECTION 4

Answer ONLY ONE question.

ALL working must be clearly shown.

7. (a) There are 60 students in the sixth form of a certain school. Mathematics is studied by 27 of them, Biology by 20 of them and 22 students study neither Mathematics nor Biology. If a student is selected at random, what is the probability that the student is studying

(i) both Mathematics and Biology? **(3 marks)**

(ii) Biology only? **(2 marks)**

(b) Two ordinary six-sided dice are thrown together. The random variable S represents the sum of the scores of their faces landing uppermost.

(i) Copy and complete the sample space diagram below.

6			9			
5		7				
4						10
3					8	
2				6		
1	2					
	1	**2**	**3**	**4**	**5**	**6**

Sample space diagram of *S*

(1 mark)

(ii) Find

a) $P(S > 9)$ **(2 marks)**

b) $P(S \leq 4)$. **(1 mark)**

(iii) Let D be the difference between the scores of the faces landing uppermost. The table below gives the probability of each possible value of d.

d	0	1	2	3	4	5
$P(D = d)$	$\frac{1}{6}$	a	$\frac{2}{9}$	b	$\frac{1}{9}$	c

Find the values of a, b and c. **(3 marks)**

(c) The aptitude scores obtained by 51 applicants for a supervisory job are summarized in the following stem and leaf diagram.

5|1 means 51

3	1 5 9
4	2 4 6 8 9
5	1 3 3 5 6 7 9
6	0 1 3 3 3 5 6 8 8 9
7	1 2 2 2 4 5 5 5 6 8 8 8 9 9
8	0 1 2 3 5 8 8 9
9	0 1 2 6

(i) Find the median and quartiles for the data given. **(4 marks)**

(ii) Construct a box-and-whisker plot to illustrate the data given and comment on the distribution of the data. **(4 marks)**

Total 20 marks

8. (a) Figure 2 below, **not drawn to scale**, shows the motion of a car with velocity, *V*, as it moves along a straight road from Point *A* to Point *B*. The time, *t*, taken to travel from Point *A* to Point *B* is 90 seconds and the distance from Point *A* to Point *B* is 1410 m.

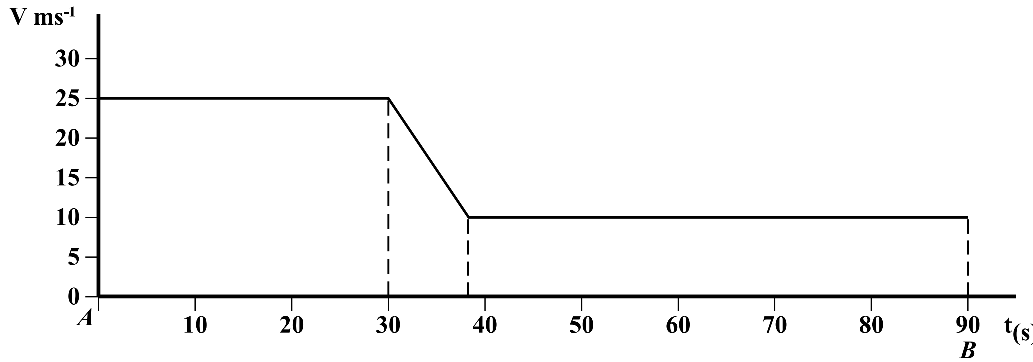

Figure 2.

(i) What distance did the car travel from Point *A* towards Point *B* before starting to decelerate? **(2 marks)**

(ii) Calculate the deceleration of the car as it goes from 25 m s^{-1} to 10 m s^{-1}. **(5 marks)**

(iii) For how long did the car maintain the speed of 10 m s^{-1}? **(1 mark)**

(iv) From Point *B*, the car decelerates uniformly, coming to rest at a Point *C* and covering a further distance of 30 m. Determine the average velocity of the car over the journey from Point *A* to Point *C*. **(2 marks)**

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(b) A particle travels along a straight line. It starts from rest at a point, P, on the line and after 10 seconds, it comes to rest at another point, Q, on the line. The velocity v m s^{-1} at time t seconds after leaving P is

$$v = 0.72t^2 - 0.096t^3 \qquad \text{for } 0 \leq t \leq 5$$

$$v = 2.4t - 0.24t^2 \qquad \text{for } 5 \leq t \leq 10$$

At maximum velocity the particle has no acceleration.

(i) Find the time when the velocity is at its maximum. **(3 marks)**

(ii) Determine the maximum velocity. **(2 marks)**

(iii) Find the distance moved by the particle from P to the point where the particle attains its maximum velocity. **(5 marks)**

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.