STRAND E: Measurement

Unit 13 *Areas*

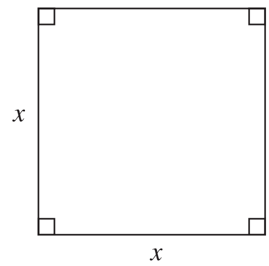
Student Text

Contents

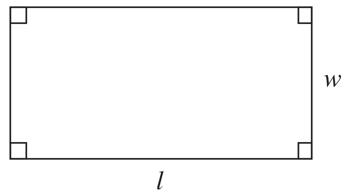
13 Areas

13.1 Squares, Rectangles and Triangles

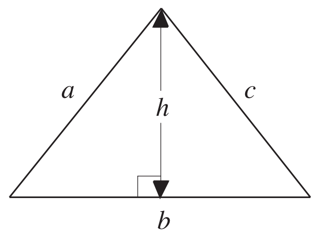
For a *square*, the area is given by $x \times x = x^2$ and the perimeter by $4x$, where x is the length of a side.



For a *rectangle*, the area is given by lw and the perimeter by $2(l + w)$, where l is the length and w the width.



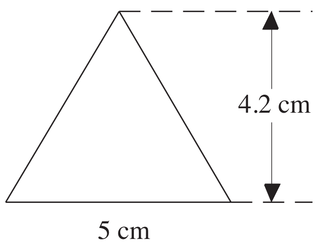
For a *triangle,* the area is given by $\frac{1}{2}bh$ and the perimeter by $a + b + c$, where b is the length of the base, h the height and a and c are the lengths of the other two sides.



Worked Example 1

Find the area of each triangle below.

(a)



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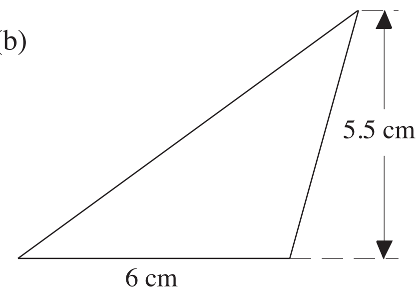


Solution

Use $\quad \text{Area} = \frac{1}{2}bh$ or $\frac{1}{2} \times \text{base} \times \text{height}$.

(a) $\quad \text{Area} = \frac{1}{2} \times 5 \times 4.2$

$= 10.5 \text{ cm}^2$

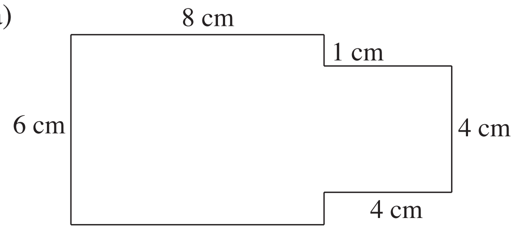
(b) $\quad \text{Area} = \frac{1}{2} \times 6 \times 5.5$

$= 16.5 \text{ cm}^2$

Worked Example 2

Find the perimeter and area of each shape below.

(a)



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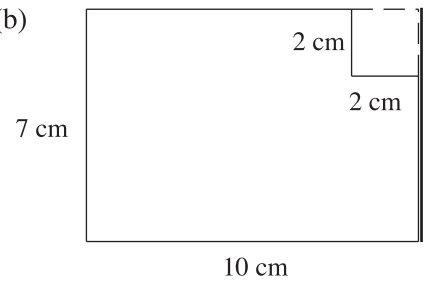
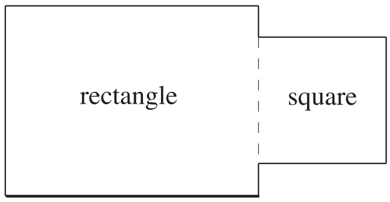


Solution

(a) The perimeter is found by adding the lengths of all the sides.

$$P = 6 + 8 + 1 + 4 + 4 + 4 + 1 + 8$$
$$= 36 \text{ cm}$$

To find the area, consider the shape split into a rectangle and a square.



$$\text{Area} = \text{Area of rectangle} + \text{Area of square}$$
$$= 6 \times 8 + 4^2$$
$$= 48 + 16$$
$$= 64 \text{ cm}^2$$

(b) Adding the lengths of the sides gives

$$P = 10 + 7 + 8 + 2 + 2 + 5$$
$$= 34 \text{ cm}$$

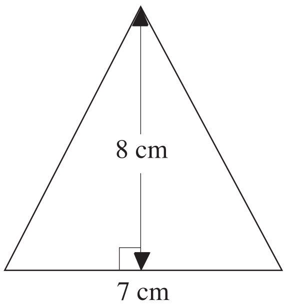
The area can be found by considering the shape to be a rectangle with a square removed from it.

$$\text{Area of shape} = \text{Area of rectangle} - \text{Area of square}$$
$$= 7 \times 10 - 2^2$$
$$= 70 - 4$$
$$= 66 \text{ cm}^2$$

Exercises

1. Find the area of each triangle.

(a)



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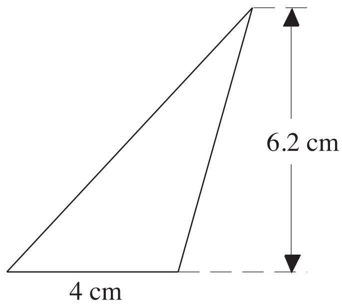

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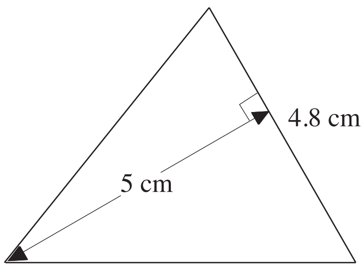

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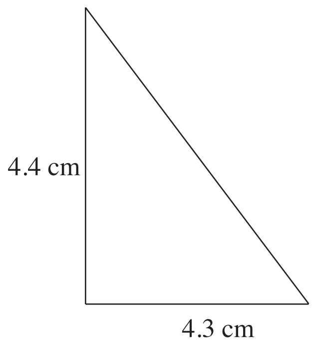

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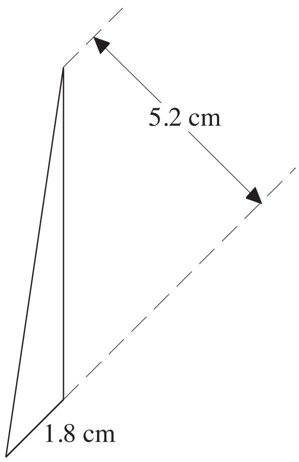

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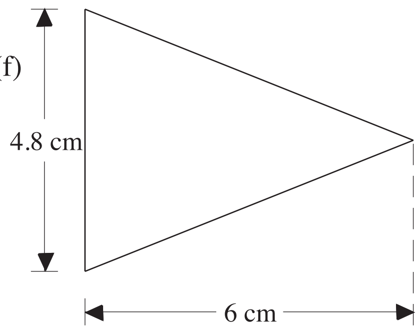
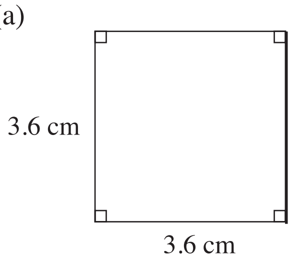


2. Find the perimeter and area of each of the following shapes.

(a)



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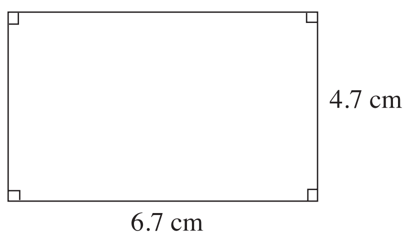

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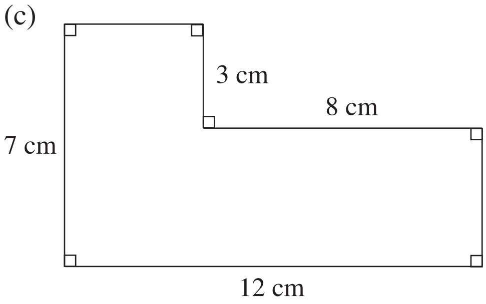

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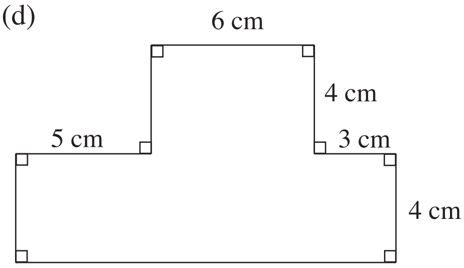

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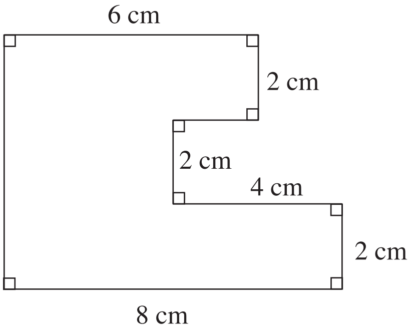

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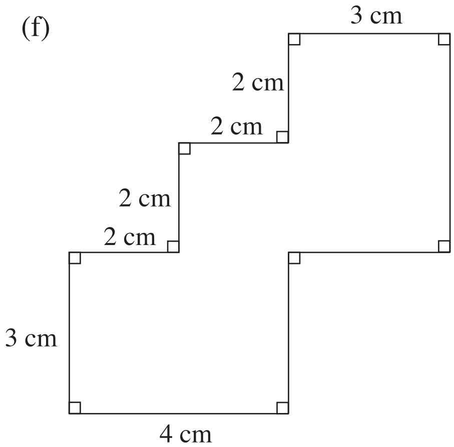
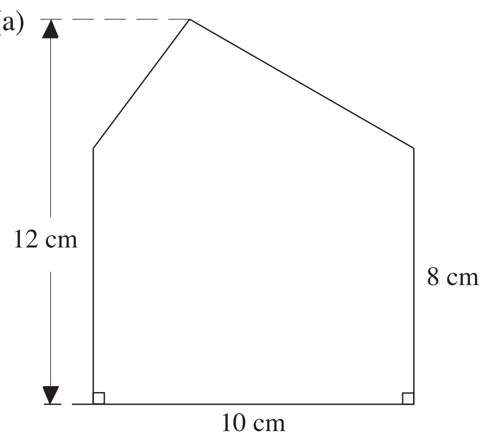
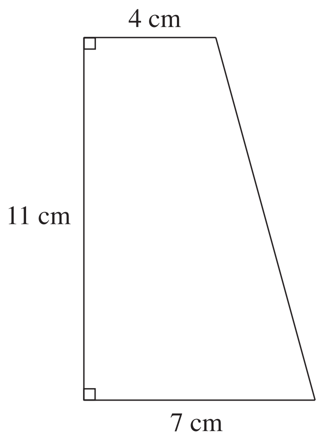


3. Find the area of each shape.

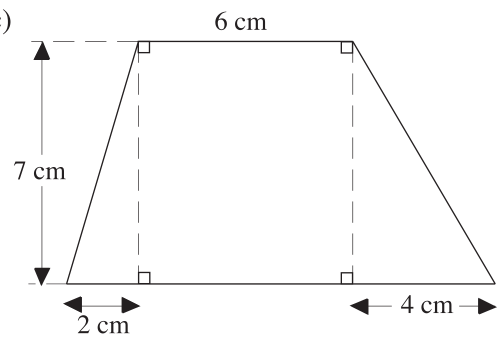
(a)



(b)



(c)



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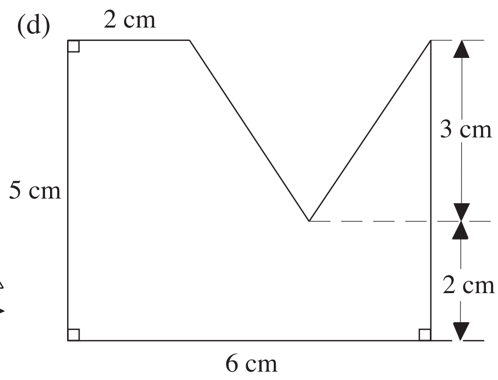
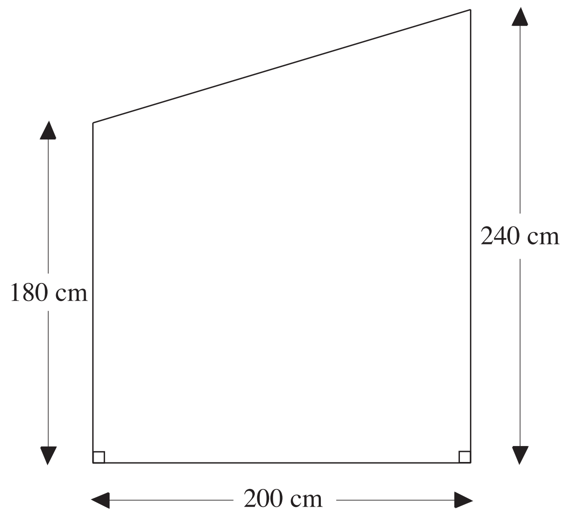


4. The diagram shows the end wall of a shed built out of concrete bricks.



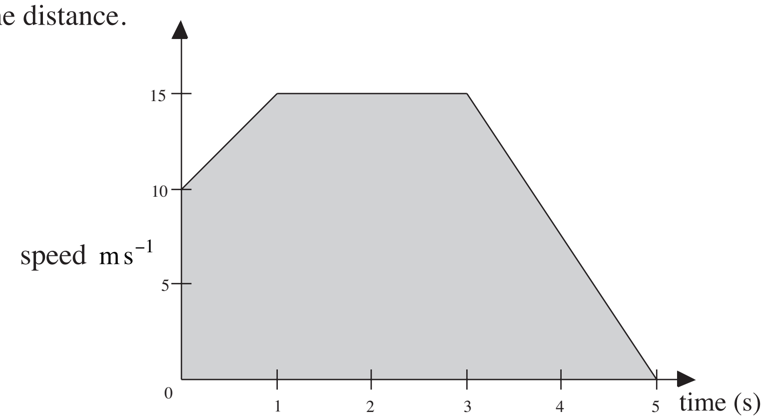
(a) Find the area of the wall.

(b) The blocks are 45 cm by 23 cm in size.

How many blocks would be needed to build the wall? (The blocks can be cut.)

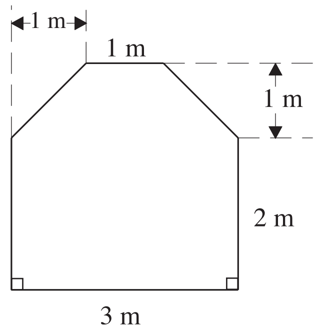
5. The shaded area on the speed time graph represents the distance travelled by a bicycle.

Find the distance.

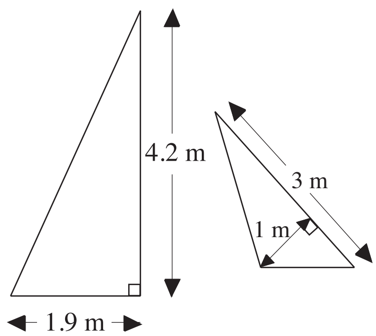


6. The plan shows the base of a storeroom.

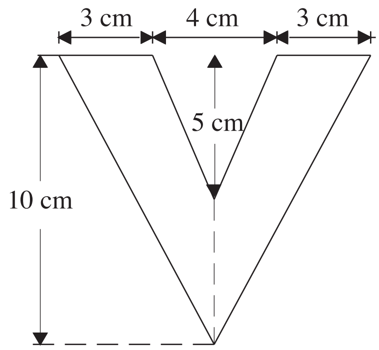
Find the area of the base.



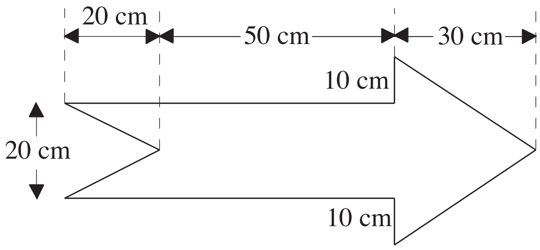
7. The diagram shows the two sails from a small sailing boat.
Find their combined area.



8. The diagram shows the letter V.
Find the area of this letter.



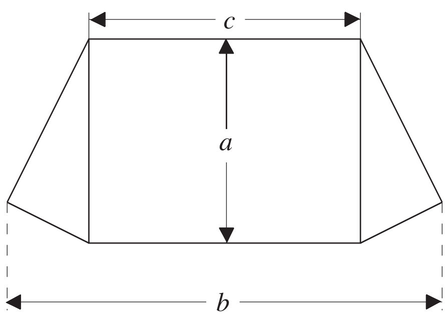
9. Find the area of the arrow shown in the diagram.



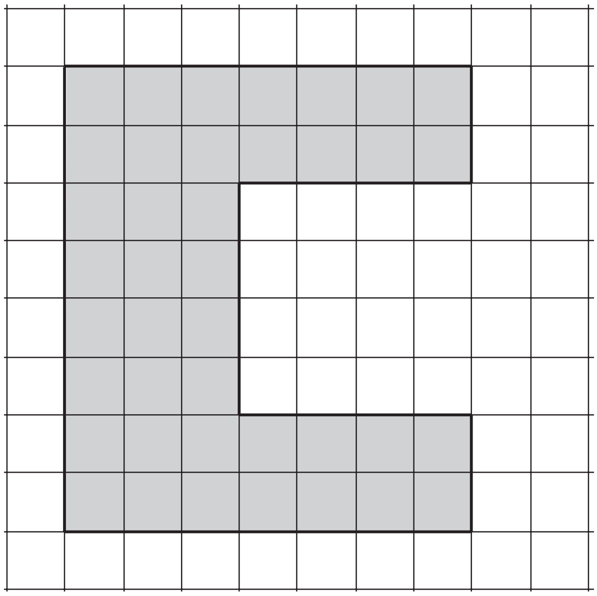
10. The diagram shows how the material required for one side of a tent is cut out.

(a) Find the area of the material shown if $b = 3.2$ m, $c = 2$ m and
(i) $a = 1.5$ m (ii) $a = 2$ m

(b) Find the area if $a = 1.6$ m, $b = 3.4$ m and $c = 2$ m.



11.

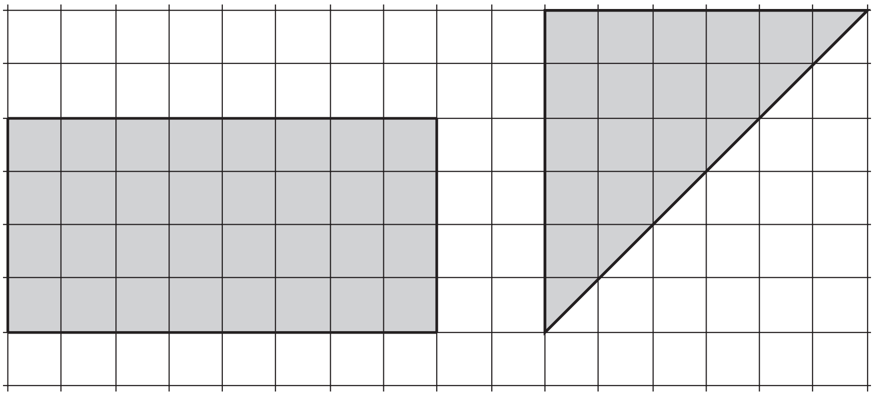


The  shape above is shaded on centimetre squared paper.

(a) Find the perimeter of this shape.

(b) Find the area of this shape.

12.



(a) What is the perimeter of the rectangle?

(b) What is the area of the triangle?

13. Work out the areas of these shapes.

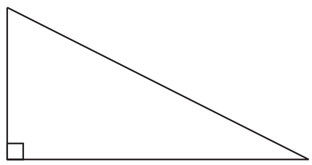
(a)

3 cm

6 cm

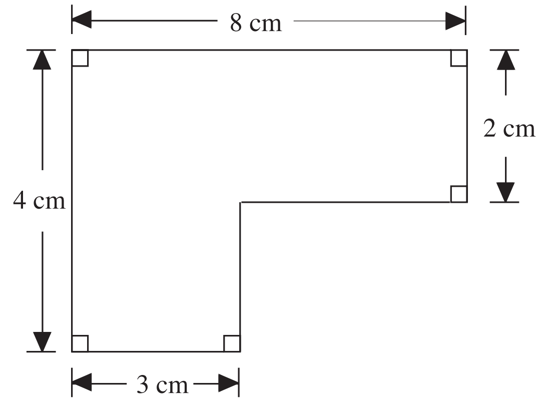
(b)

5 cm

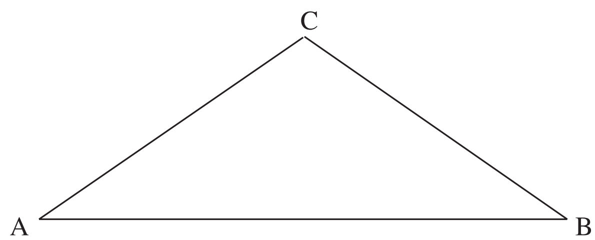


12 cm

14. Calculate the area of this shape.

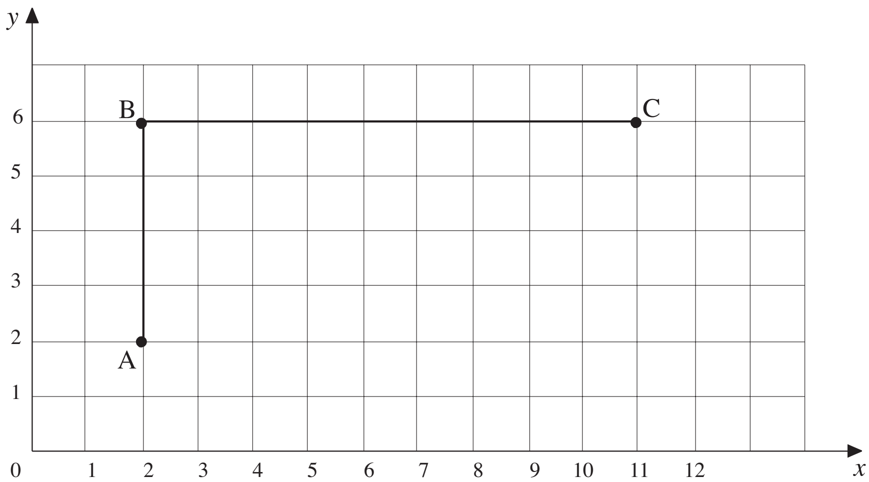


15.



By making and using appropriate measurements, calculate the area of triangle ABC in square centimetres. State the measurements that you have made and show your working clearly.

16.



(a) Write down the coordinates of the midpoint of AC.

(b) Copy the diagram and mark and label a point D so that ABCD is a rectangle.

(c) (i) Find the perimeter of the rectangle ABCD.

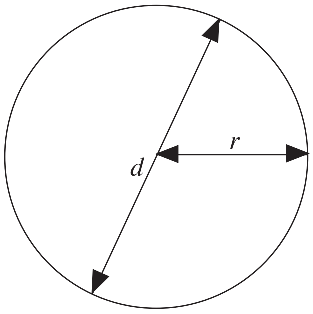
(ii) Find the area of the rectangle ABCD.

(d) The rectangle has reflective (line) symmetry.

Describe another type of symmetry that it has.

13.2 Area and Circumference of Circles

The *circumference* of a circle can be calculated using



$$C = 2\pi r \quad \text{or} \quad C = \pi d$$

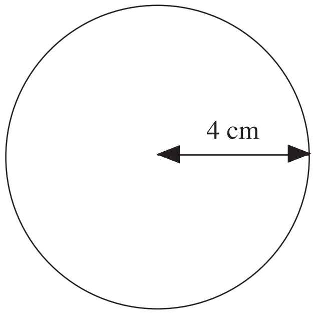
where r is the radius and d the diameter of the circle.

The *area* of a circle is found using

$$A = \pi r^2 \quad \text{or} \quad A = \frac{\pi d^2}{4}$$

Worked Example 1

Find the circumference and area of this circle.



Solution

The circumference is found using $C = 2\pi r$, which in this case gives

$$C = 2\pi \times 4$$

$$= 25.1 \text{ cm} \qquad \text{(to one decimal place)}$$

The area is found using $A = \pi r^2$, which gives

$$A = \pi \times 4^2$$

$$= 50.3 \text{ cm}^2 \qquad \text{(to one decimal place)}$$

Worked Example 2

Find the radius of a circle if:

(a) its circumference is 32 cm, (b) its area is 14.3 cm^2.

Solution

(a) Using $C = 2\pi r$ gives

$$32 = 2\pi r$$

and dividing by 2π gives

$$\frac{32}{2\pi} = r$$

so that

$$r = 5.09 \text{ cm} \quad \text{(to 2 decimal places)}$$

(b) Using $A = \pi r^2$ gives

$$14.3 = \pi r^2$$

Dividing by π gives

$$\frac{14.3}{\pi} = r^2$$

Then taking the square root of both sides gives

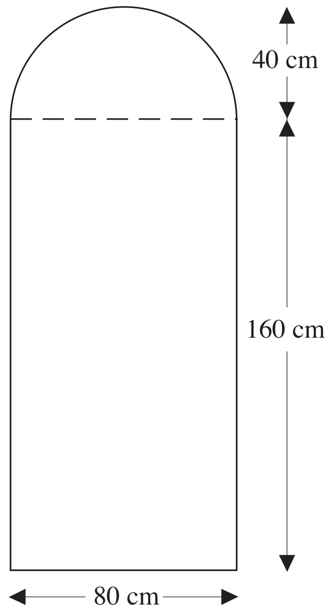
$$\sqrt{\frac{14.3}{\pi}} = r$$

so that

$$r = 2.13 \text{ cm} \quad \text{(to 2 decimal places)}$$

Worked Example 3

Find the area of the door shown in the diagram.
The top part of the door is a semicircle.



Solution

First find the area of the rectangle.

$$\text{Area} = 80 \times 160$$
$$= 12\,800 \text{ cm}^2$$

Then find the area of the semicircle.

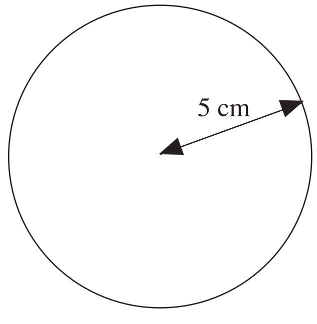
$$\text{Area} = \frac{1}{2} \times \pi \times 40^2$$
$$= 2513 \text{ cm}^2$$

$$\text{Total area} = 12\,800 + 2513$$
$$= 15\,313 \text{ cm}^2 \quad \text{(to the nearest cm}^2\text{)}$$

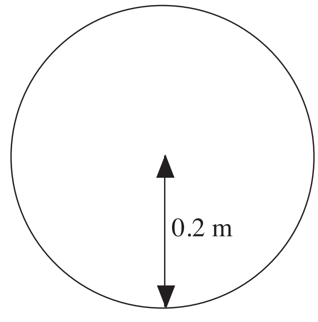
Exercises

1. Find the circumference and area of each of the following circles.

(a)



(b)



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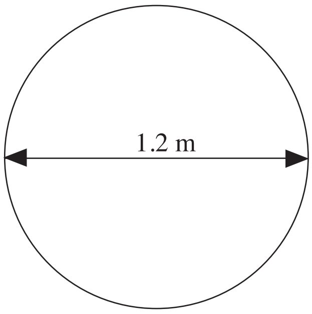
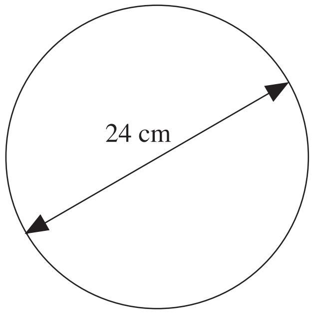


(d)



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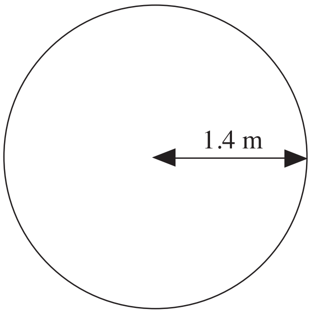

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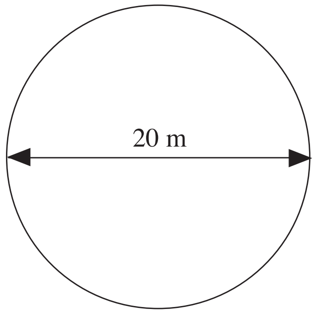


2. Find the radius of the circle which has:

   (a) a circumference of 42 cm,

   (b) a circumference of 18 cm,

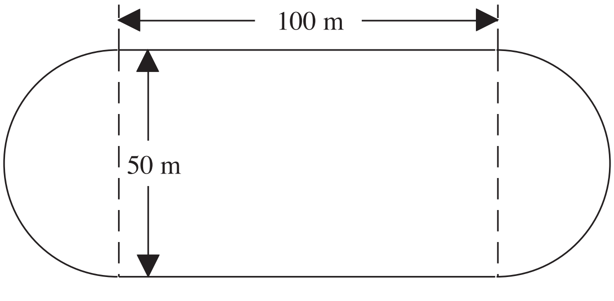
   (c) an area of 69.4 cm^2,

   (d) an area of 91.6 cm^2.

3. The diagram shows a running track.

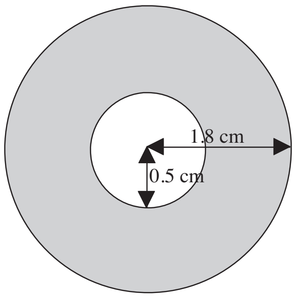
   (a) Find the length of one complete circuit of the track.

   (b) Find the area enclosed by the track.

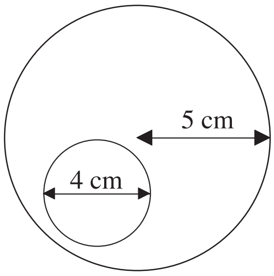


4. A pipe-washer has an outer radius of 1.8 cm and an inner radius of 0.5 cm.

   Find the area that has been shaded in the diagram, to the nearest cm^2.



5. An egg, fried perfectly, can be thought of as a circle (the yolk) within a larger circle (the white).



(a) Find the area of the smaller circle that represents the surface of the yolk.

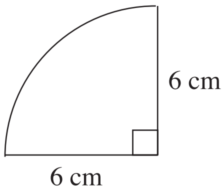
(b) Find the area of the surface of the whole egg.

(c) Find the area of the surface of the white of the egg, to the nearest cm^2 .

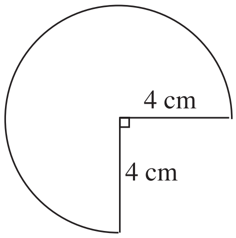
6. The shapes shown below were cut out of card, ready to make cones.

Find the area of each shape.

(a)



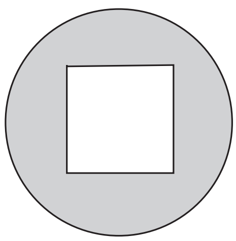
(b)



7. A circular hole with diameter 5 cm is cut out of a rectangular metal plate of length 10 cm and width 7 cm. Find the area of the plate when the hole has been cut out.

8. Find the area of the wasted material if two circles of radius 4 cm are cut out of a rectangular sheet of material that is 16 cm long and 8 cm wide.

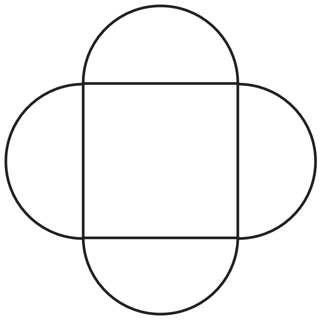
9. A square hole is cut in a circular piece of card to create the shape shown.



(a) Find the shaded area of the card if the radius of the circle is 5.2 cm and the sides of the square are 4.8 cm.

(b) Find the radius of the circle if the shaded area is 50 cm^2 and the square has sides of length 4.2 cm.

10. Four semicircles are fixed to the sides of a square as shown in the diagram, to form a design for a table top.



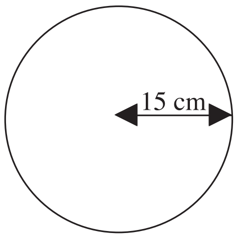
(a) Find the area of the table top if the square has sides of length 1.5 m.

(b) Find the length of the sides of the square and the total area of the table top if the area of each semicircle is 1 m^2.

11. The radius of a circle is 8 cm.

Work out the area of the circle.
(Use $\pi = 3.14$ or the π button on your calculator.)

12. A circle has a radius of 15 cm.



Calculate the area of the circle.

Take π to be 3.14 or use the π key on your calculator.

13. Lecia does a sponsored bicycle ride.

Each wheel of her bicycle is of radius 25 cm.

(a) Calculate the circumference of one of the wheels

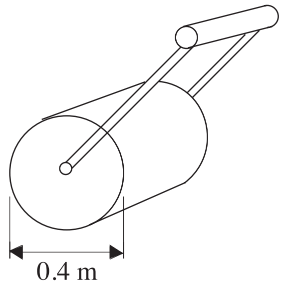
(b) She cycles 50 km. How many revolutions does a wheel make during the sponsored ride?

14. The diameter of a garden roller is 0.4 m.

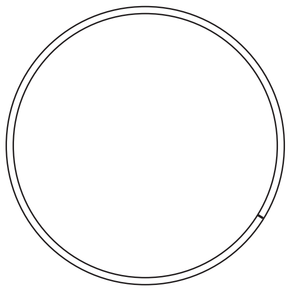
The roller is used on a path of length 20 m.

Calculate how many times the roller rotates when rolling the length of the path once.

Take π to be 3.14 or use the π key on your calculator.



15.



A piece of rope is 12 metres long. It is laid on the ground in a circle, as shown in the diagram.

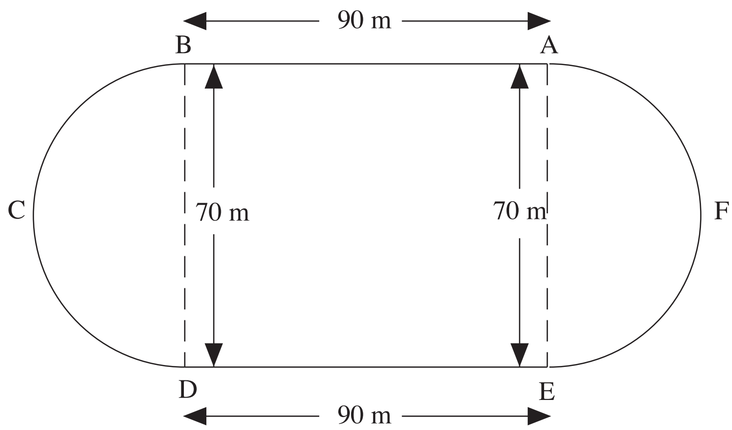
(a) Using 3.14 as the value of π, calculate the diameter of the circle.

(b) Explain briefly how you would check the answer to part (a) mentally.

The cross-section of the rope is a circle of radius 1.2 cm.

(c) Calculate the area of the cross-section.

16.



The diagram shows a running track.

BA and DE are parallel and straight. They are each of length 90 metres.

BCD and EFA are semicircular. They each have a diameter of length 70 metres.

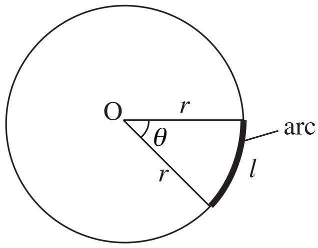
Using $\pi = \frac{22}{7}$, calculate

(a) the perimeter of the track,

(b) the total area inside the track.

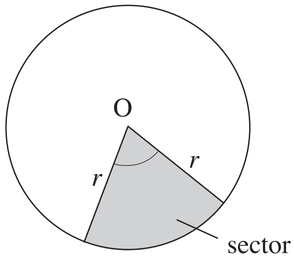
13.3 Sector Areas and Arc Lengths

A part of the circumference of a circle is called an *arc*. If the angle subtended by the arc at the centre of the circle is θ then the arc length l is given by



$$\boxed{l = \frac{\theta}{360°} \times 2\pi r}$$

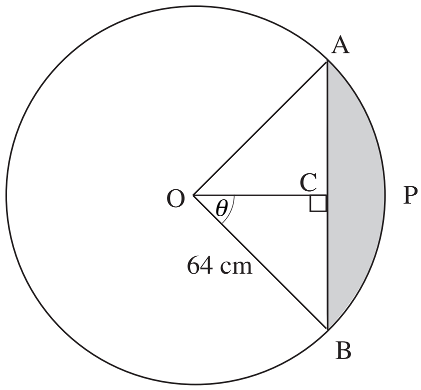
The region between the two radii and the arc is called a *sector* of the circle. The area of the sector of the circle is



$$\boxed{A = \frac{\theta}{360°} \times \pi r^2}$$

Worked Example 1

The shaded area shows a segment of a circle of radius 64 cm.
The length of the chord AB is 100 cm.



(a) Find the angle θ, to 2 d.p.

(b) Find the area of triangle OAB.

(c) Find the area of the sector of the circle with angle 2θ.

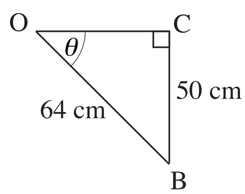
(d) Find the area of the segment shaded in the figure.

Solution

(a) If $AB = 100$ cm then, by symmetry, $BC = 50$ cm.

$$\sin\theta = \frac{50}{64}$$

$$\theta = 51.38°$$



(b) The area of the triangle OAB is $\frac{1}{2} \times 64^2 \times \sin 2\theta = 1997 \text{ cm}^2$

(This is using the result area of triangle $= \frac{1}{2} ab \sin\theta$, which is covered in Unit 34.)

(c) The sector has area $\frac{(2 \times 51.38°)}{360°} \times \pi \times 64^2 = 3673 \text{ cm}^2$

(d) The segment has area $3673 - 1997 = 1676 \text{ cm}^2$.

Worked Example 2

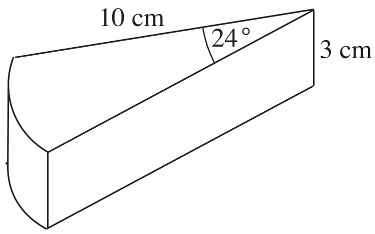
A wooden door wedge is in the shape of a sector of a circle of radius 10 cm with angle $24°$ and constant thickness 3 cm.

Find the volume of wood used in making the wedge.

Solution

The area of the top face of the wedge is the area of a sector of radius 10 cm and angle $24°$.

$$\text{Area} = \frac{24°}{360°} \times \pi \times 10^2 = \frac{20\pi}{3} = 20.94 \text{ cm}^2$$



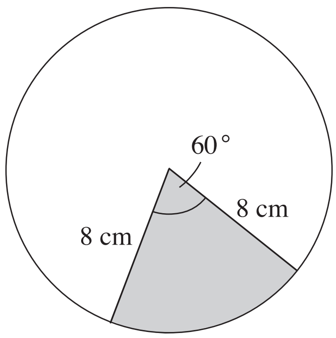
The volume of the wedge

$$= \text{Area} \times 3 = 20\pi = 62.83 \text{ cm}^3$$

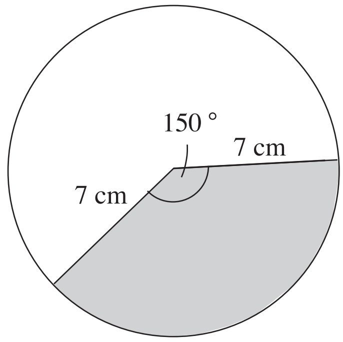
Exercises

1. Find the area of the shaded regions in the following figures.

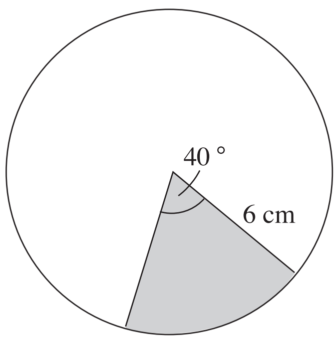
(a)



(b)

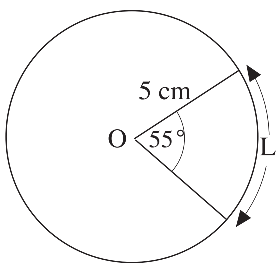


(c)

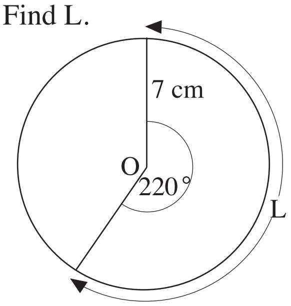


2. Find the values of the unknowns marked in the following.

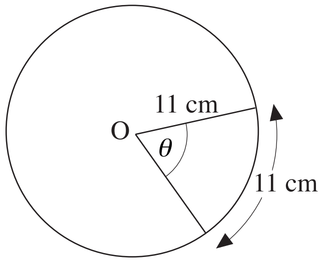
(a) Find L.



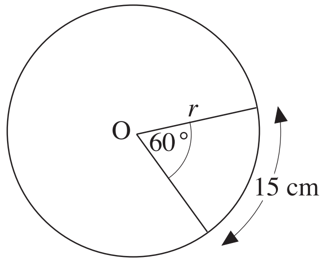
(b) Find L.



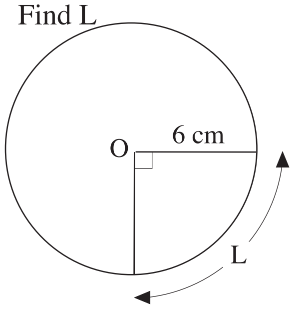
(c) Find θ.



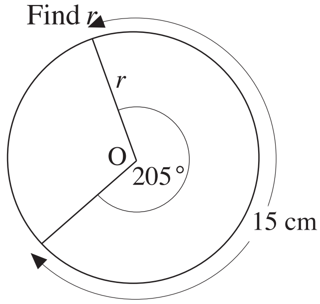
(d) Find r.



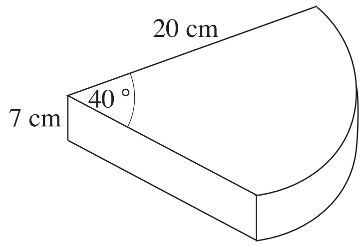
(e) Find L



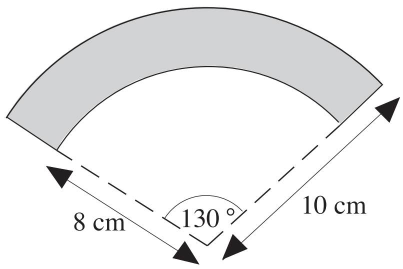
(f) Find r.



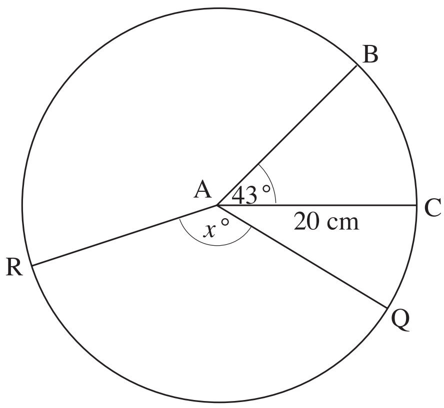
3. A cake is made in the shape of a sector of a circle with size shown. The thickness of the cake is 7 cm. The top and edges are to be covered with marzipan of thickness $\frac{1}{2}$ cm. Find the volume of marzipan needed.



4. Find the area of the shaded region shown.



5.



BAC is a sector of a circle, radius 20 cm, whose centre is at A.

Angle BAC = 43°.

(a) Calculate the area of the sector BAC.

(b) The area of sector QAR is 450 cm^2.

Angle QAR is $x°$.

Calculate the value of x.

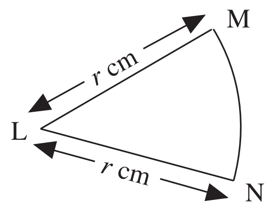
(c) The area of the sector MLN of another circle, centre L, is 600 cm^2.

The total perimeter of the sector is 100 cm.

It can be shown that the radius, r cm, of the sector satisfies the equation

$$r^2 - 50r + 600 = 0$$

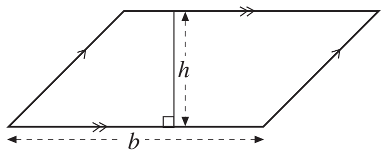
Find the values of r which satisfy this equation.



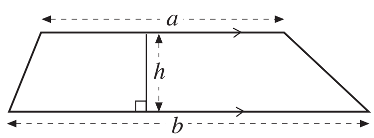
13.4 Areas of Parallelograms, Trapeziums, Kites and Rhombuses

The formulae for calculating the areas of these shapes are:

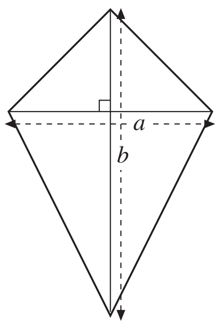
Parallelogram $\boxed{A = bh}$



Trapezium $\boxed{A = \frac{1}{2}(a + b)h}$



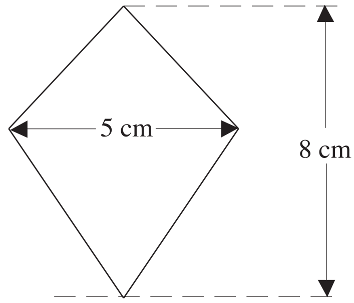
Kite $\boxed{A = \frac{1}{2}ab}$



The area of a *rhombus* can be found using either the formula for a kite or the formula for a parallelogram.

Worked Example 1

Find the area of this kite.



Solution

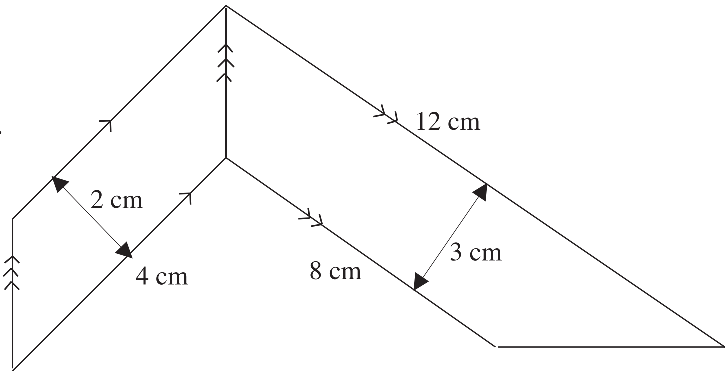
Using the formula $A = \frac{1}{2}ab$

with $a = 5$ and $b = 8$ gives

$$A = \frac{1}{2} \times 5 \times 8$$
$$= 20 \text{ cm}^2$$

Worked Example 2

Find the area of this shape.



Solution

The shape is made up of a parallelogram and a trapezium.

$$\text{Area of parallelogram} = 2 \times 4$$
$$= 8 \text{ cm}^2$$

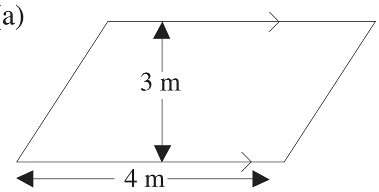
$$\text{Area of trapezium} = \frac{1}{2}(8 + 12) \times 3$$
$$= 30 \text{ cm}^2$$

$$\text{Total area} = 8 + 30$$
$$= 38 \text{ cm}^2$$

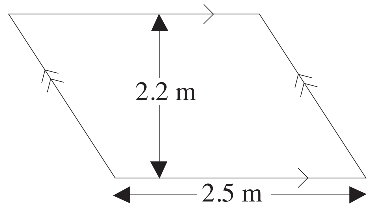
Exercises

Find the area of each of the following shapes.

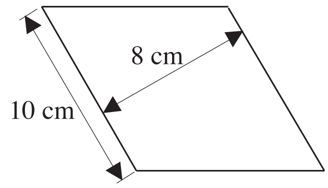
1. (a)



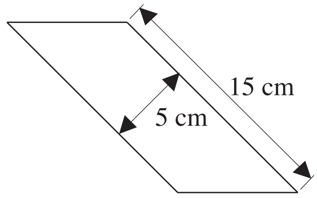
(b)



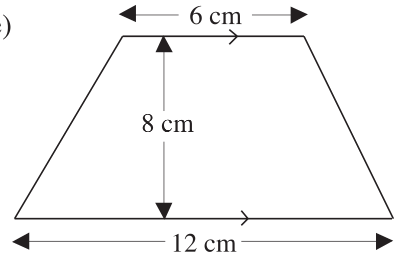
(c)



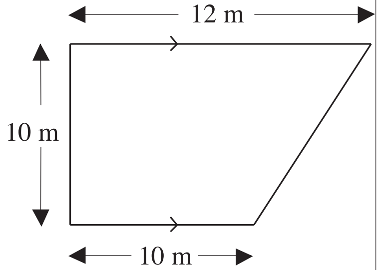
(d)



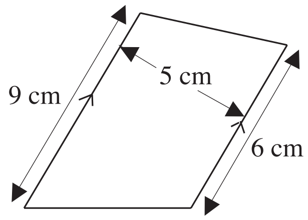
(e)



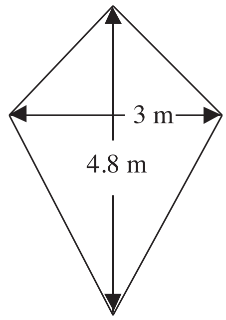
(f)



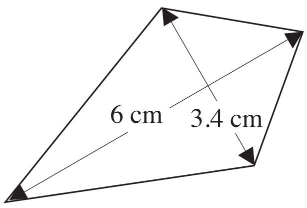
(g)



(h)

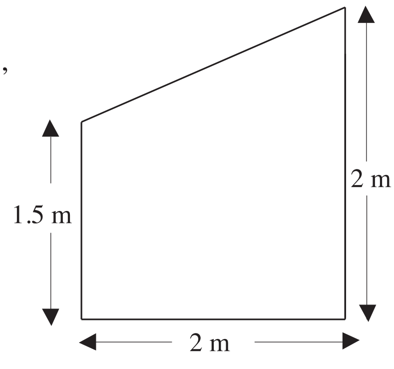


(i)



2. The diagram shown the end wall of a wooden shed,

(a) Find the area of this end of the shed.



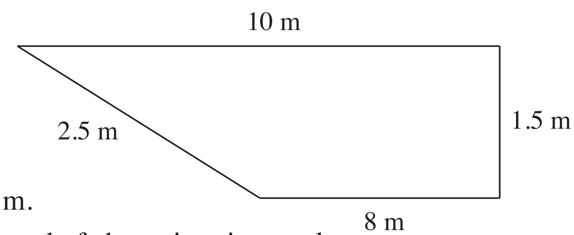
The other end of the shed is identical. The sides are made up of two rectangles of length 3 m.

(b) Find the area of each side of the shed.

(c) Find the total area of the walls of the shed.

3. The diagram shows the vertical side of a swimming pool.

(a) Find the area of the side of the pool.



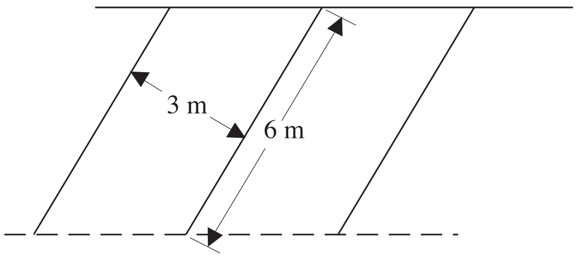
The width of the swimming pool is 4 m.

(b) Find the area of the rectangular end of the swimming pool.

(c) Find the area of the horizontal base of the pool.

(d) Find the total area of the sides and horizontal base of the pool.

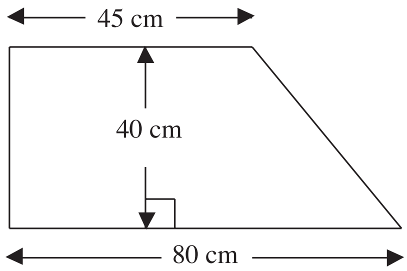
4. In a car park, spaces are marked out in parallelograms.



Find the area of each parking space.

5. The diagram shows a window of a car.

Find the area of the window.

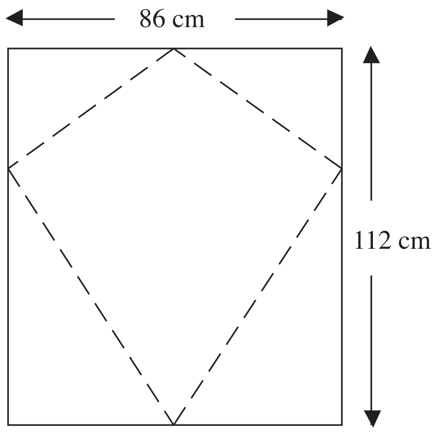


6. A kite is cut out of a sheet of plastic as shown.

(a) Find the area of the kite.

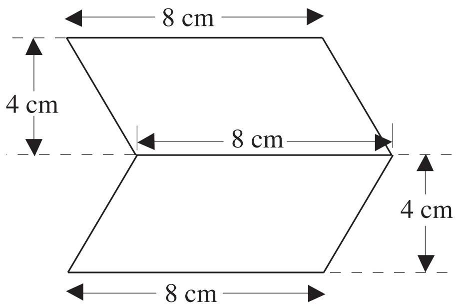
(b) Find the area of the plastic that would be wasted.

(c) Would you obtain similar results if you cut a kite out of a rectangle of plastic with dimensions 140 cm by 80 cm?

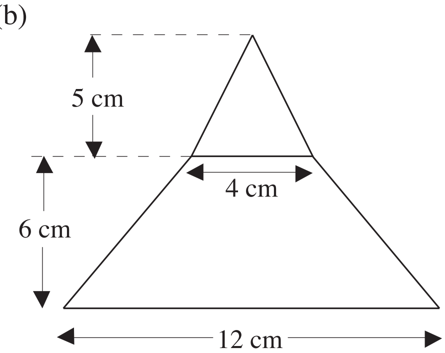


7. Find the area of each of the following shapes.

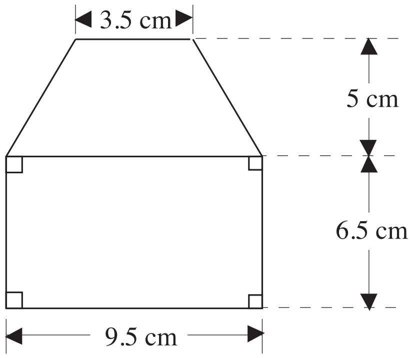
(a)



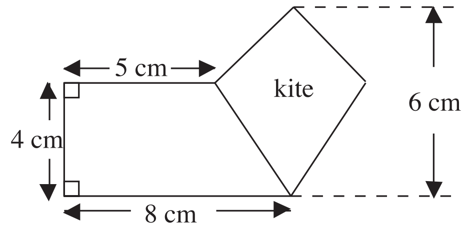
(b)



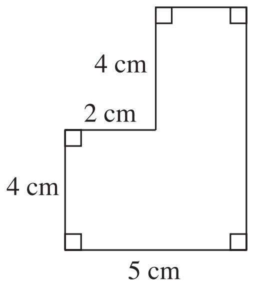
(c)



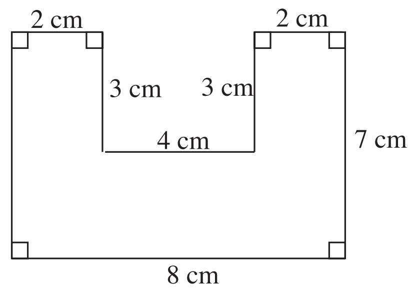
(d)



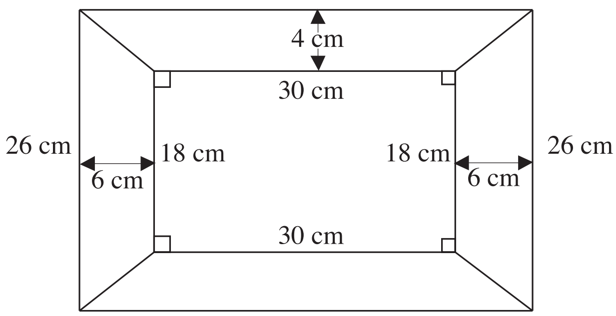
(e)



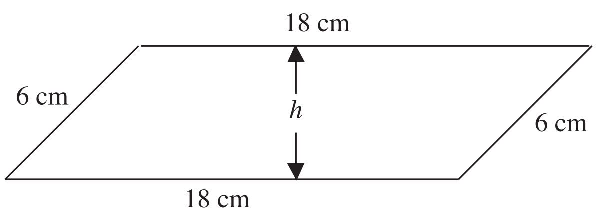
(f)



8. A simple picture frame is made by joining four trapezium shaped strips of wood. Find the area of each trapezium and the total area of the frame.



9. Four rods are joined together to form a parallelogram.



(a) Find the area of the parallelogram if:

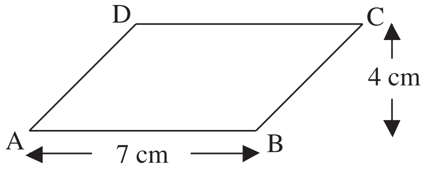
(i) $h = 2$ cm (ii) $h = 4$ cm (iii) $h = 5$ cm

(b) Can h be higher than 6 cm?

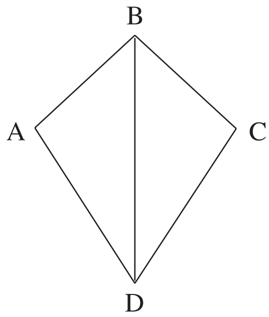
(c) What is the maximum possible area of the parallelogram?

10. (a) Find the area of parallelogram ABCD.

    (b) Find the area of the triangle ABC.



11.

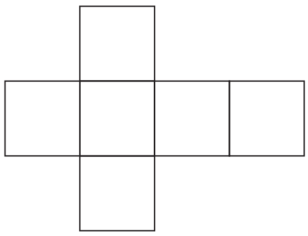


Not to scale

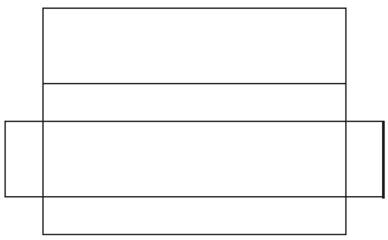
Why is the area of the kite ABCD equal to twice the area of the triangle ABD?

13.5 Surface Area

The *net* of a cube can be used to find its surface area.

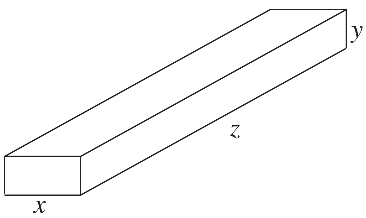


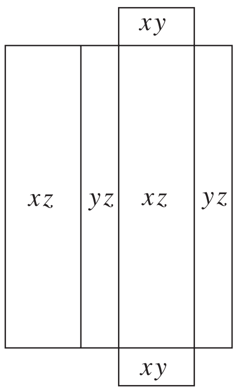
The net is made up of 6 squares, so the surface area will be 6 times the area of one square. If x is the length of the sides of the cube its surface area will be $6x^2$.



This diagram shows the net for a cuboid. To find the surface area the area of each of the 6 rectangles must be found and then added to give the total.

If x, y and z are the lengths of the sides of the cuboid, then the area of the rectangles in the net are as shown here.

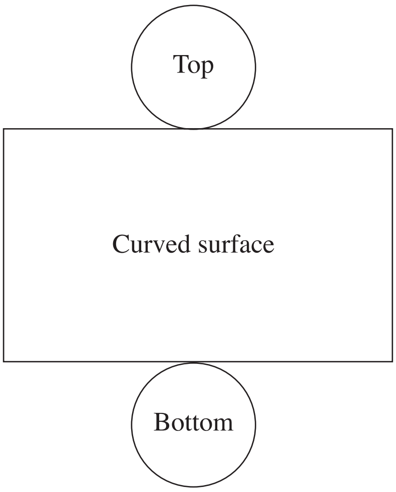


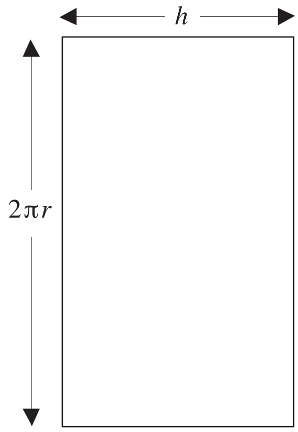


The total surface area of the cuboid is then given by

$$\boxed{A = 2xy + 2xz + 2yz}$$

To find the surface area of a cylinder, consider how a cylinder can be broken up into three parts, the top, bottom and curved surface.





The areas of the top and bottom are the same and each is given by πr^2.

The curved surface is a rectangle. The length of one side is the same as the circumference of the circles, $2\pi r$, and the other side is simply the height of the cylinder, h. So the area is $2\pi r h$.

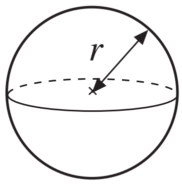
The total surface area of the cylinder is

$$\boxed{2\pi r^2 + 2\pi r h}$$

Another important result is the *surface area of a sphere*.

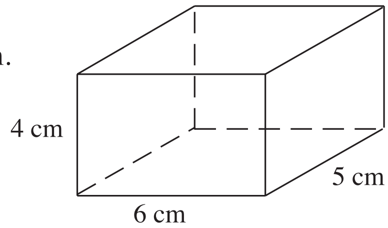
For a sphere with radius *r*, the surface area is given by the formula

$$\boxed{4\pi r^2}$$



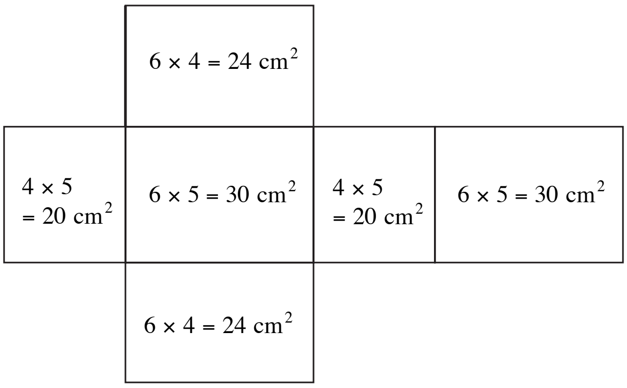
Worked Example 1

Find the surface area of the cuboid shown in the diagram.



Solution

The diagram shows the net of the cuboid and the areas of the rectangles that it contains.



Using the net, the total surface area is given by

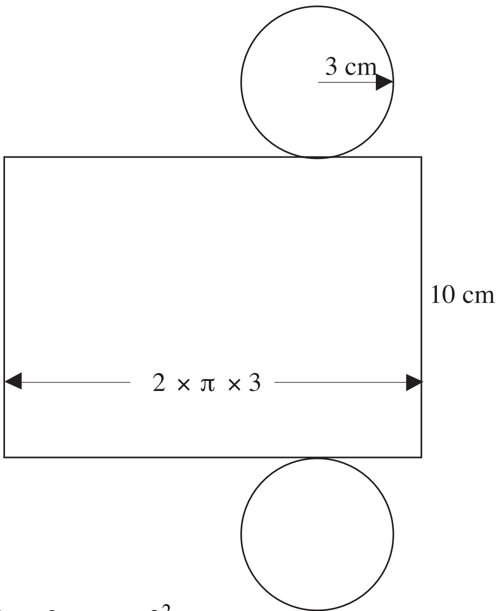
$$A = 2 \times 20 + 2 \times 30 + 2 \times 24$$
$$= 148 \text{ cm}^2$$

Worked Example 2

Cans are made out of aluminium sheets, and are cylinders of radius 3 cm and height 10 cm. Find the area of aluminium needed to make one can.

Solution

The diagram shows the two circles and the rectangle from which cans will be made.



The rectangle has one side as 10 cm, the height of the cylinder and the other side is $2 \times \pi \times 3$ cm, the circumference of the top and bottom.

The area of the rectangle is $10 \times 2 \times \pi \times 3$

The area of each circle is $\pi \times 3^2$

So the total surface area is $A = 10 \times 2 \times \pi \times 3 + 2 \times \pi \times 3^2$

$$= 245.04 \text{ cm}^2 \text{ (to 2 d.p.)}$$

Worked Example 3

A beach tennis ball has radius 4 cm. What is its surface area, to the nearest cm?

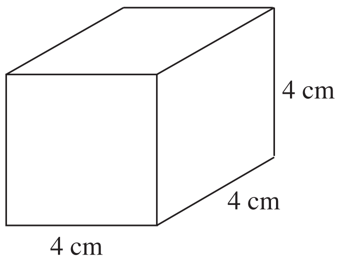
Solution

$$
\begin{aligned}
\text{Surface area} &= 4\pi r^2 \text{ cm}^2 \\
&= 4\pi 4^2 \text{ cm}^2 \\
&= 64\pi \text{ cm}^2 \\
&= 201 \text{ cm}^2 \text{ to the nearest cm}^2
\end{aligned}
$$

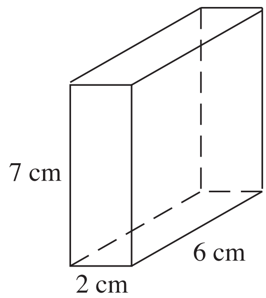
Exercises

1. Find the surface area of each of the following cubes or cuboids.

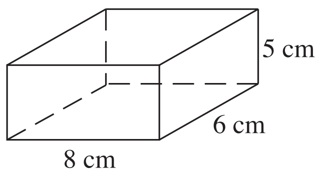
(a)



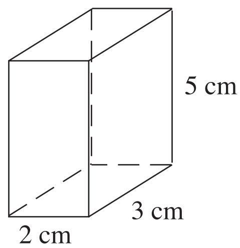
(b)



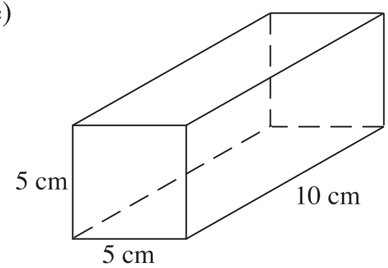
(c)



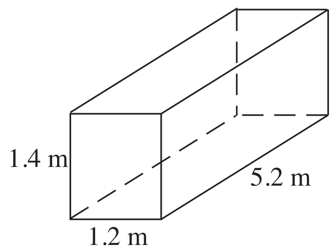
(d)



(e)

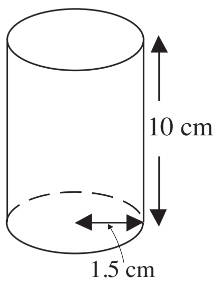


(f)

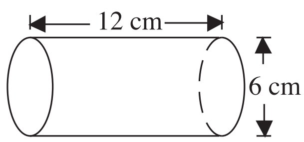


2. Find the total surface area of each cylinder shown below.

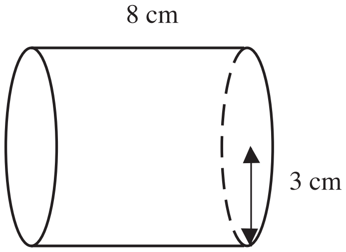
(a)



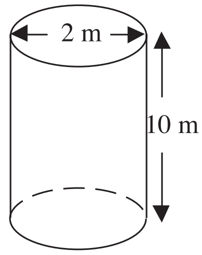
(b)



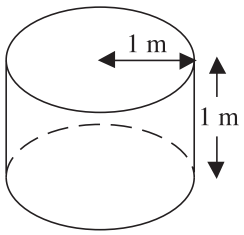
(c)



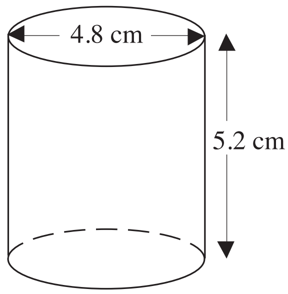
(d)



(e)



(f)



3. A groundsman uses a roller to compact the surface of a cricket pitch. The roller consists of a cylinder of radius 30 cm and width 70 cm.

   (a) Find the area of ground that the roller covers as the cylinder completes 1 rotation.

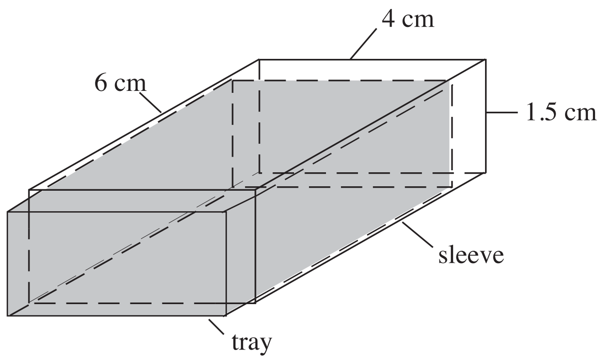
   (b) If the roller is pulled 5 m, what area of ground does the roller cover?

4. A matchbox consists of a tray that slides into a sleeve. If the tray and sleeve have the same dimensions and no material is used up in joins, find:

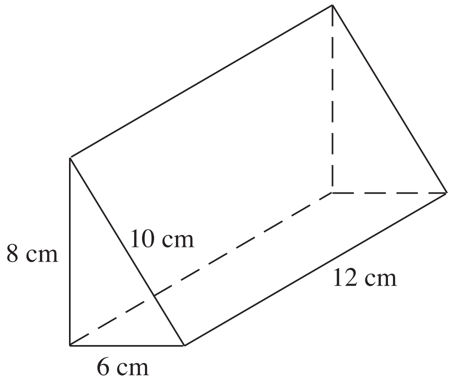
   (a) the area of cardboard needed to make the tray,

   (b) the area of cardboard needed to make the sleeve,

   (c) the total area of the cardboard needed to make the matchbox.

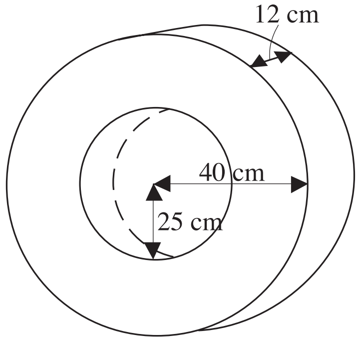


5. Draw a net of the prism shown in the diagram opposite and use it to find the surface area of the prism.

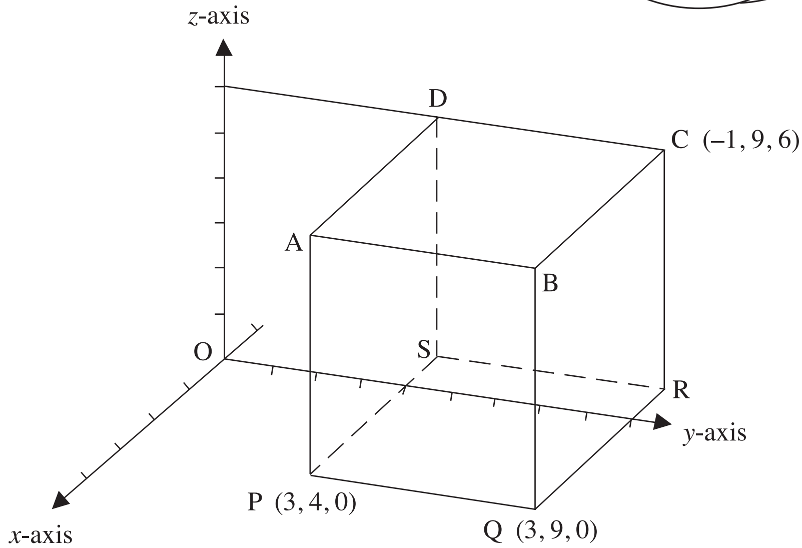


6. A car tyre can be thought of as a hollow cylinder with a hole cut out of the centre.

   Find the surface area of the entire exterior of the tyre.



7.



   The diagram shows a cuboid.

   The co-ordinates of P are $(3, 4, 0)$.
   The co-ordinates of Q are $(3, 9, 0)$.
   The co-ordinates of C are $(-1, 9, 6)$.

   (a) Write down the (x, y, z) co-ordinates

      (i) of R   (ii) of B.

   (b) Write down the lengths of each of the following edges of the cuboid.

      (i) PQ   (ii) QR.

   (c) Calculate the total surface area of the cuboid.

8. A beach ball has diameter 30 cm. What is its surface area, to the neareast tenth of m^2 ?

Information

Smaller animals have more surface area compared to their volume than larger animals. Because of this, smaller animals tend to lose water and body heat more easily than larger animals. Children have $2\frac{1}{2}$ times as much surface area compared to volume as adults. Thus children are more prone to dehydration and hypothermia.