

# STRAND I: Geometry and Trigonometry

## Unit 34 *Pythagoras' Theorem and Trigonometric Ratios*

### Student Text

#### Contents

##### Section

- |      |   |
|------|---|
| 34.1 | Pythagoras' Theorem                       |
| 34.2 | Further Work With Pythagoras' Theorem     |
| 34.3 | Sine, Cosine and Tangent                  |
| 34.4 | Finding Lengths in Right Angled Triangles |
| 34.5 | Finding Angles in Right Angled Triangles  |

# 34 Pythagoras' Theorem and Trigonometric Ratios

## 34.1 Pythagoras' Theorem

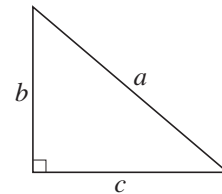
Pythagoras' Theorem gives a relationship between the lengths of the sides of a right angled triangle.

*Pythagoras' Theorem* states that:

In any right angled triangle, the area of the square on the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares on the other two sides (the two sides that meet at the right angle).

For the triangle shown opposite,

$$a^2 = b^2 + c^2$$



### Note

The longest side of a right angled triangle is called the *hypotenuse*.



### Proof

Draw a square of side  $b + c$ , as shown opposite. Join up the points PQ, QR, RS, SP as shown, to give a quadrilateral, PQRS.

In fact, PQRS is a square as each side is equal to  $a$  (as the four triangles are congruent) and at the point P,

$$x + \text{angle SPQ} + y = 180^\circ$$

But we know that  $x + y = 90^\circ$ , so

$$\text{angle SPQ} = 90^\circ$$

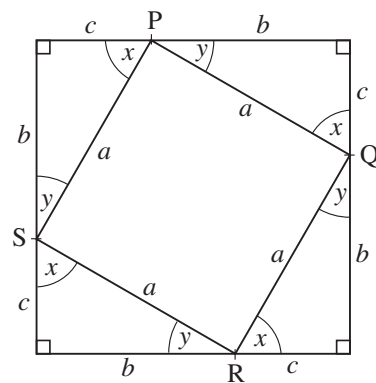
Similarly for the other three angles in PQRS. Thus PQRS is a square, and equating areas,

$$a^2 + 4 \times \left(\frac{1}{2}bc\right) = (b + c)^2$$

$$a^2 + 2bc = b^2 + 2bc + c^2$$

Hence

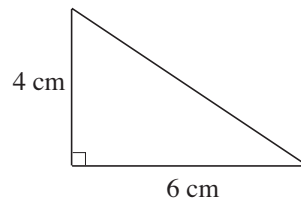
$$a^2 = c^2 + b^2$$





### Worked Example 1

Find the length of the hypotenuse of the triangle shown in the diagram. Give your answer correct to 2 decimal places.



### Solution

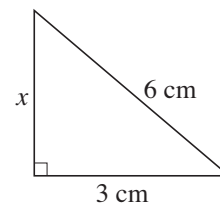
As this is a right angled triangle, Pythagoras' Theorem can be used. If the length of the hypotenuse is  $a$ , then  $b = 4$  and  $c = 6$ .

$$\begin{aligned} \text{So } a^2 &= b^2 + c^2 \\ a^2 &= 4^2 + 6^2 \\ a^2 &= 16 + 36 \\ a^2 &= 52 \\ a &= \sqrt{52} \\ &= 7.2 \text{ cm} \quad (\text{to one decimal place}) \end{aligned}$$



### Worked Example 2

Find the length of the side of the triangle marked  $x$  in the diagram.



### Solution

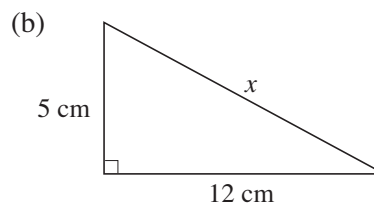
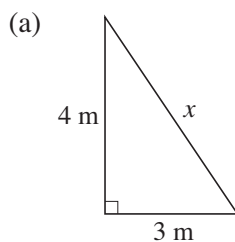
As this is a right angled triangle, Pythagoras' Theorem can be used. Here the length of the hypotenuse is 6 cm, so writing  $a = 6$  cm and  $c = 3$  cm with  $b = x$ , we have

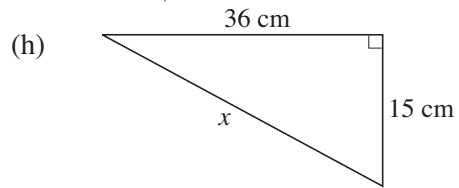
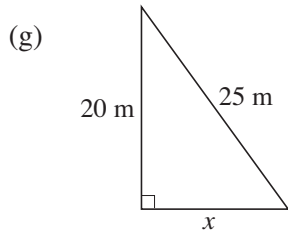
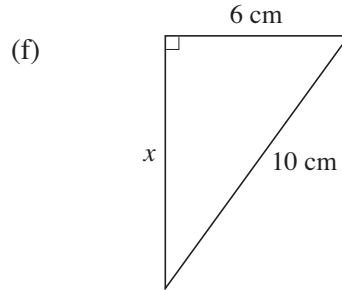
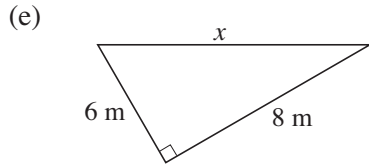
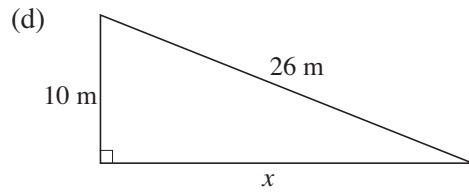
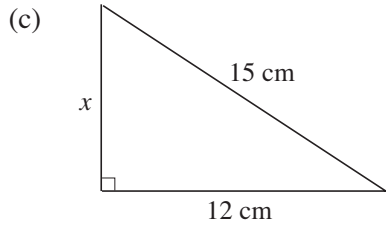
$$\begin{aligned} a^2 &= b^2 + c^2 \\ 6^2 &= x^2 + 3^2 \\ 36 &= x^2 + 9 \\ 36 - 9 &= x^2 \\ 27 &= x^2 \\ \sqrt{27} &= x \\ x &= 5.2 \text{ cm} \quad (\text{to one decimal place}) \end{aligned}$$



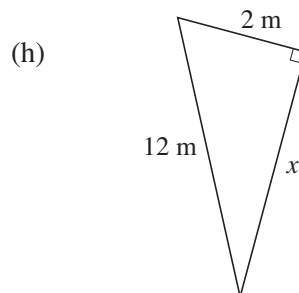
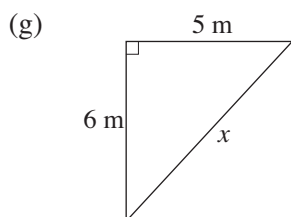
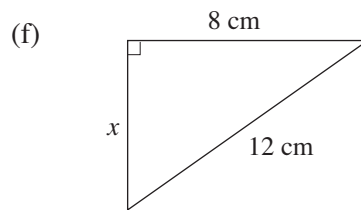
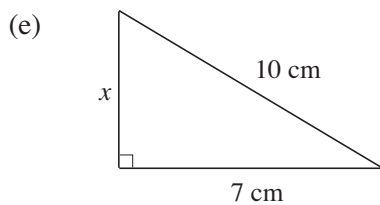
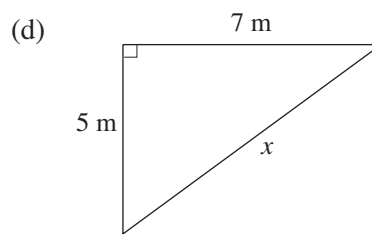
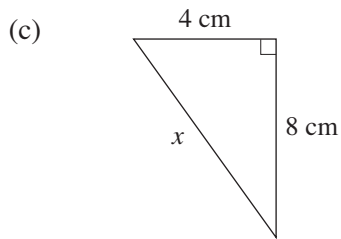
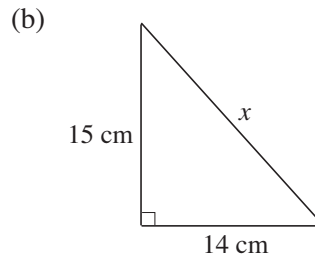
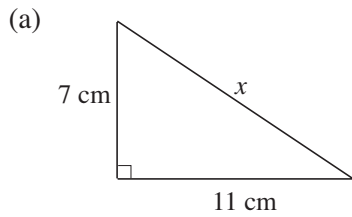
### Exercises

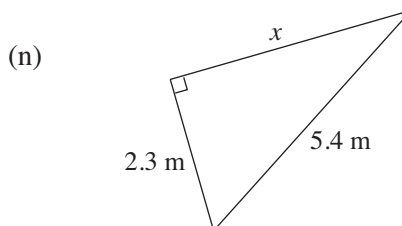
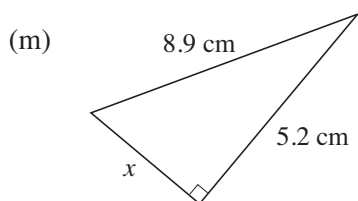
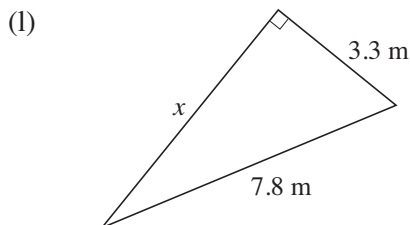
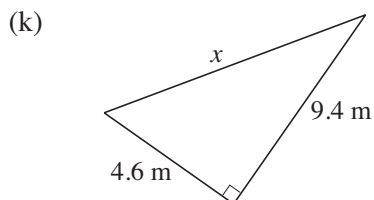
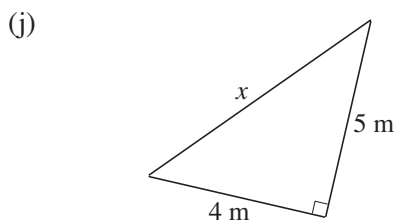
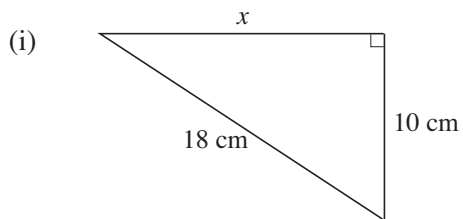
1. Find the length of the side marked  $x$  in each triangle.





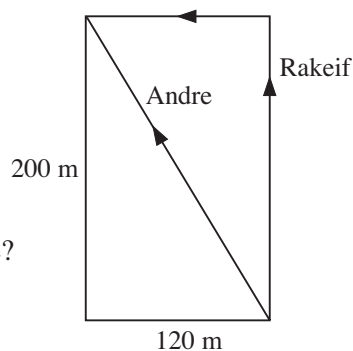
2. Find the length of the side marked  $x$  in each triangle. Give your answers correct to 2 decimal places.



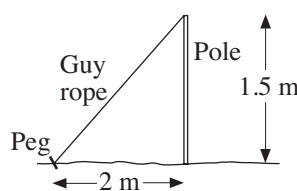


3. Andre runs diagonally across a school field, while Rakeif runs around the edge.

- (a) How far does Rakeif run?
- (b) How far does Andre run?
- (c) How much further does Rakeif run than Andre?



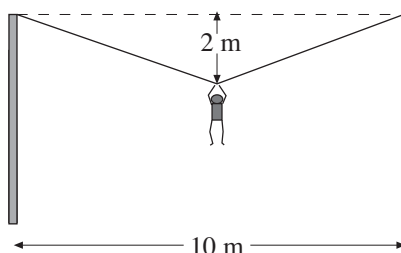
4. A guy rope is attached to the top of a tent pole, at a height of 1.5 metres above the ground, and to a tent peg 2 metres from the base of the pole. How long is the guy rope?



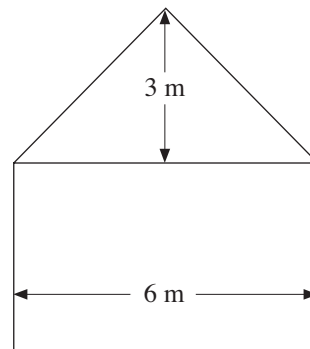
5. Deneen is 1.4 metres tall. At a certain time her shadow is 2 metres long. What is the distance from the top of her head to the tip of her shadow?

6. A rope of length 10 metres is stretched from the top of a pole 3 metres high until it reaches ground level. How far is the end of the line from the base of the pole?

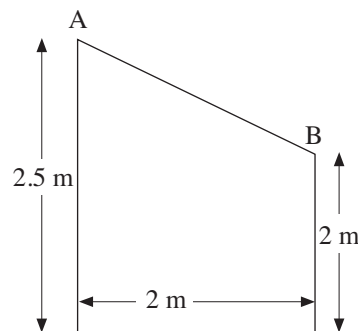
7. A rope is fixed between two trees that are 10 metres apart. When a child hangs on to the centre of the rope, it sags so that the centre is 2 metres below the level of the ends. Find the length of the rope.



8. The roof on a house that is 6 metres wide peaks at a height of 3 metres above the top of the walls. Find the length of the sloping side of the roof.

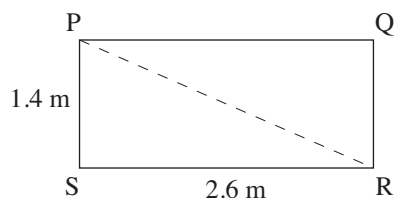


9. The picture shows a shed. Find the length, AB, of the roof.



10. Rohan walks 3 km east and then 10 km north.
- How far is he from his starting point?
  - He then walks east until he is 20 km from his starting point. How much further east has he walked

11. Jerine is building a shed. The base PQRS of the shed should be a rectangle measuring 2.6 metres by 1.4 metres.



To check that or if the base is rectangular, Jerine has to measure the diagonal PR.

- Calculate the length of PR when the base is rectangular. You *must* show all your working.
- When building the shed Jerine finds angle  $\text{PSR} > 90^\circ$ . She measures PR. Which of the following statements is *true*?
 

X: PR is greater than it should be.

Y: PR is less than it should be.

Z: PR is the right length.



## Information

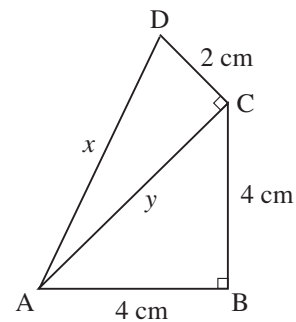
The Greeks, (in their analysis of the arcs of circles) were the first to establish the relationships or ratios between the sides and the angles of a right angled triangle. The Chinese also recognised the ratios of sides in a right angled triangle and some survey problems involving such ratios were quoted in *Zhou Bi Suan Jing*. It is interesting to note that sound waves are related to the sine curve. This discovery by Joseph Fourier, a French mathematician, is the essence of the electronic musical instrument developments today.

## 34.2 Further Work with Pythagoras' Theorem



### Worked Example 1

Find the length of the side marked  $x$  in the diagram.



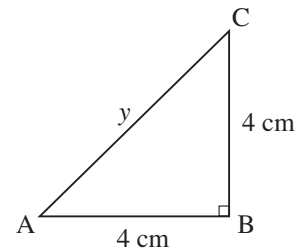
### Solution

First consider triangle ABC. The unknown length of the hypotenuse has been marked  $y$ .

$$y^2 = (4 \text{ cm})^2 + (4 \text{ cm})^2$$

$$y^2 = 16 \text{ cm}^2 + 16 \text{ cm}^2$$

$$y^2 = 32 \text{ cm}^2$$



Triangle ACD can now be considered, using the value for  $y^2$ .

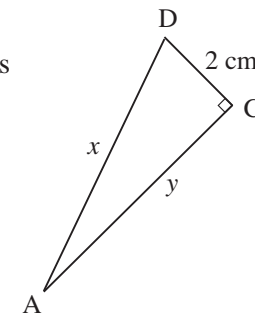
From the triangle,  $x^2 = y^2 + 2^2$ , and using  $y^2 = 32 \text{ cm}^2$  gives

$$x^2 = 32 \text{ cm}^2 + 4 \text{ cm}^2$$

$$x^2 = 36 \text{ cm}^2$$

$$x = \sqrt{36} \text{ cm}^2$$

$$x = 6 \text{ cm}$$



### Note

When finding the side  $x$ , it is not necessary to find  $\sqrt{32}$ , but to simply use  $y^2 = 32 \text{ cm}^2$ .



### Worked Example 2

Find the value of  $x$  as shown on the diagram.



### Solution

Using Pythagoras' Theorem gives

$$13^2 = (2x)^2 + (3x)^2$$

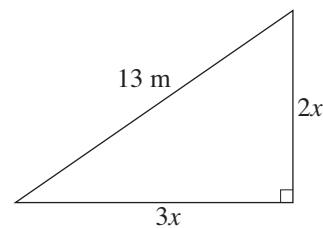
$$169 = 4x^2 + 9x^2 \quad (\text{since } (2x)^2 = 2^2 x^2 = 4x^2)$$

$$169 = 13x^2$$

$$13 = x^2$$

$$x = \sqrt{13}$$

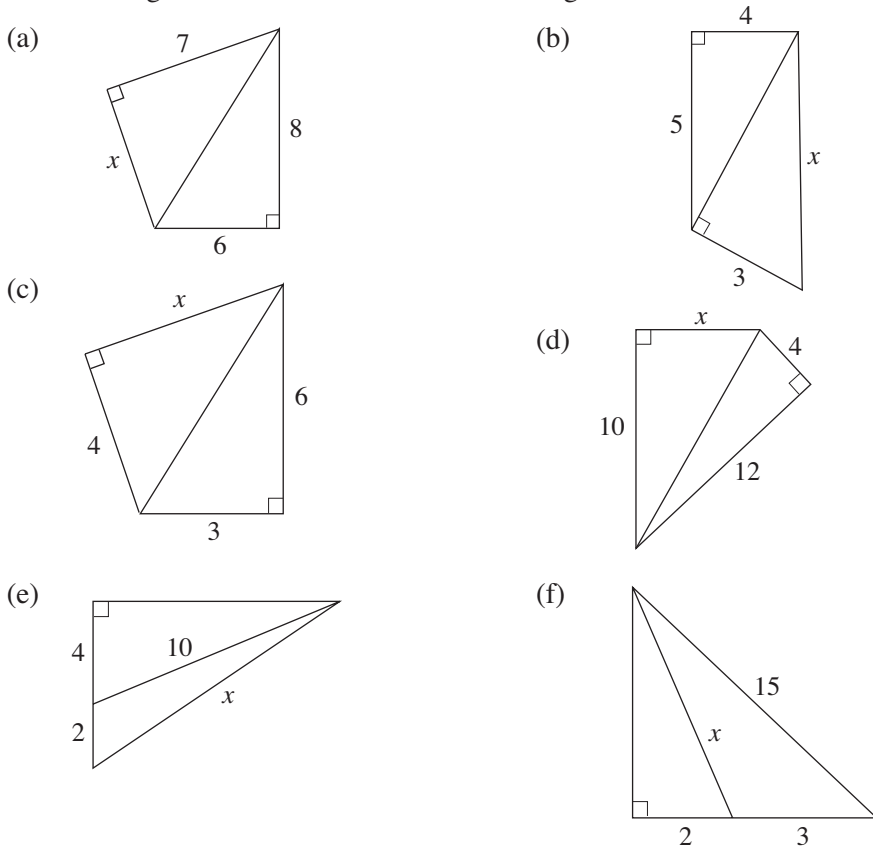
$$= 3.61 \text{ m} \quad (\text{to 2 decimal places})$$



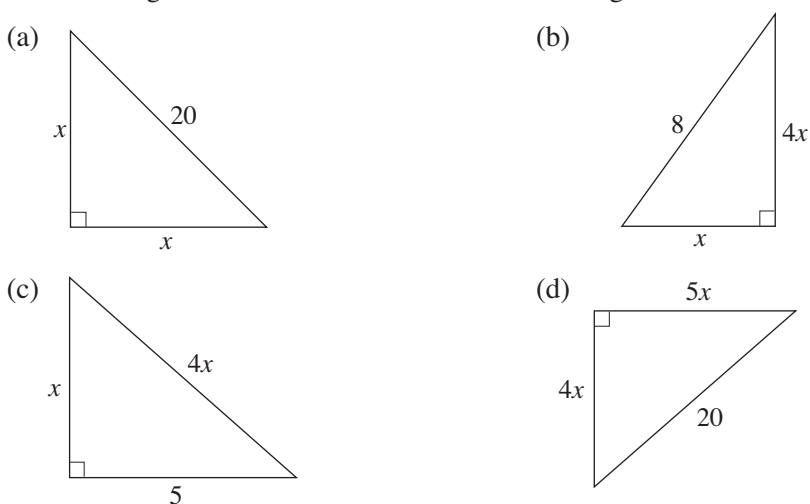


## Exercises

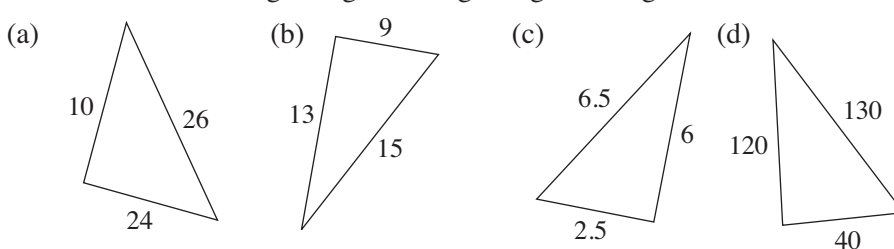
1. Find the length of the side marked  $x$  in each diagram.



2. Find the length of the side marked  $x$  in the following situations.



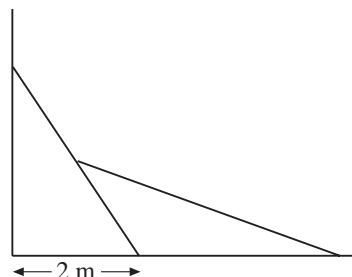
3. Which of the following triangles are right angled triangles?



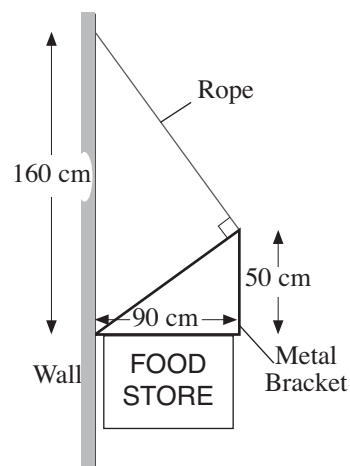


4. A ladder of length 4 metres leans against a vertical wall. The foot of the ladder is 2 metres from the wall. A plank that has a length of 5 metres rests on the ladder, so that one end is halfway up the ladder.

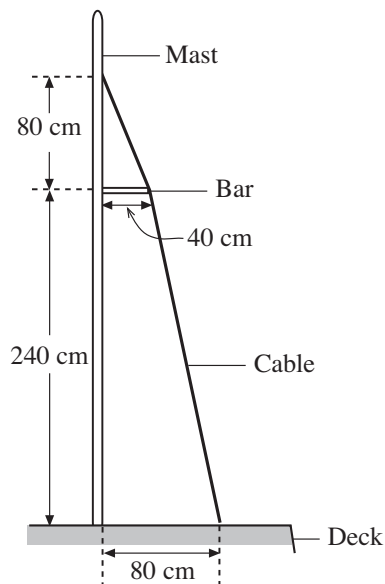
- How high is the top of the ladder?
- How high is the top of the plank?
- How far is the bottom of the plank from the wall?



5. The diagram shows how the sign that hangs over a food store is suspended by a rope and a triangular metal bracket. Find the length of the rope.

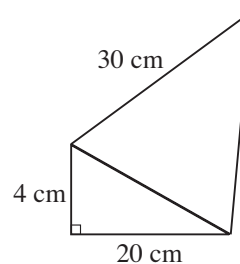


6. The diagram shows how a cable is attached to the mast of a sailing dingy. A bar pushes the cable out away from the mast. Find the total length of the cable.

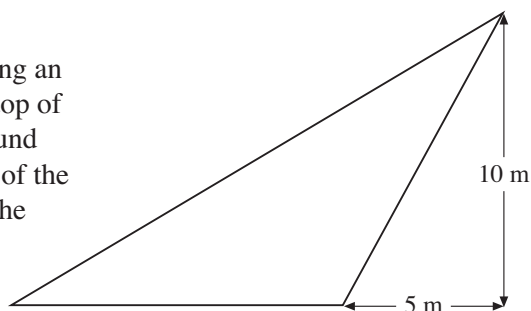


7. A helicopter flies in a straight line until it reaches a point 20 km east and 15 km north of its starting point. It then turns through  $90^\circ$  and travels a further 10 km.
- How far is the helicopter from its starting point?
  - If the helicopter turned  $90^\circ$  the other way, how far would it end up from its starting point?

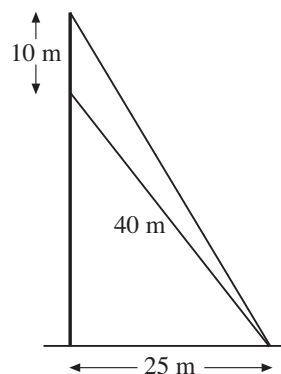
8. A cone is placed on a wedge. The dimensions of the wedge are shown in the diagram. The cone has a slant height of 30 cm. Find the height of the cone.



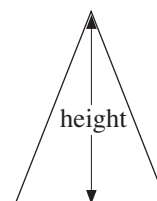
9. A simple crane is to be constructed using an isosceles triangular metal frame. The top of the frame is to be 10 metres above ground level and 5 metres away from the base of the crane, as shown in the diagram. Find the length of each side of the triangle.



10. A thin steel tower is supported on one side by two cables. Find the height of the tower and the length of the longer cable.



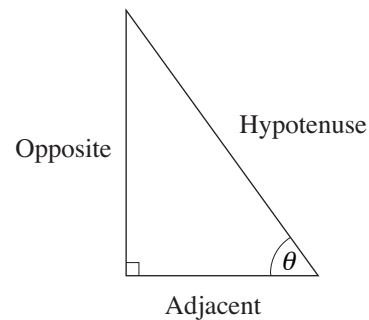
11. An isosceles triangle has two sides of length 8 cm and one of length 4 cm. Find the height of the triangle and its area.



12. Find the area of each the equilateral triangles that have sides of lengths
- 8 cm
  - 20 cm
  - 2 cm

## 34.3 Sine, Cosine and Tangent

When working in a right angled triangle, the longest side is known as the *hypotenuse*. The other two sides are known as the *opposite* and the *adjacent*. The adjacent is the side next to a marked angle, and the opposite side is opposite this angle.



For a right angled triangle, the *sine*, *cosine* and *tangent* of the angle  $\theta$  are defined as:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

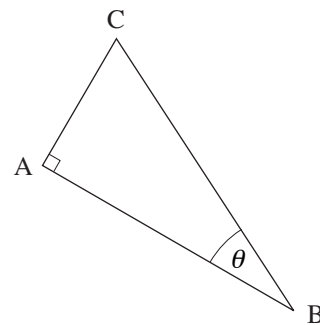
$\sin \theta$  will always have the same value for any particular angle, regardless of the size of the triangle. The same is true for  $\cos \theta$  and  $\tan \theta$ .



### Worked Example 1

For the triangle and angle shown, state which side is:

- the hypotenuse
- the adjacent
- the opposite.



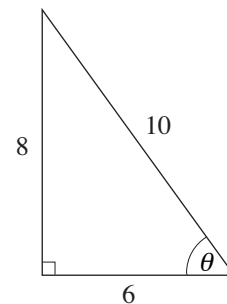
### Solution

- The hypotenuse is the longest side, which for this triangle is CB.
- The adjacent is the side that is next to the angle  $\theta$ , which for this triangle is AB.
- The opposite side is the side that is opposite the angle  $\theta$ , which for this triangle is AC.



### Worked Example 2

Write down the values of  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$  for the triangle shown. Then use a calculator to find the angle in each case.



### Solution

$$\begin{aligned} \text{First, } \quad \text{opposite} &= 8 \\ &\text{adjacent} &= 6 \\ &\text{hypotenuse} &= 10 \end{aligned}$$

$$\begin{aligned} \sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} & \cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} & \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{8}{10} & &= \frac{6}{10} & &= \frac{8}{6} \\ &= 0.8 & &= 0.6 & &= \frac{4}{3} \end{aligned}$$

Using a calculator gives  $\theta = 53.1^\circ$  (correct to 1 decimal place) in each case.



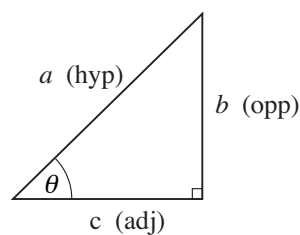
### Note

In the triangle opposite, we know that

$$\sin \theta = \frac{b}{a}, \quad \cos \theta = \frac{c}{a}$$

Thus

$$\begin{aligned} \sin^2 \theta + \cos^2 \theta &= \left(\frac{b}{a}\right)^2 + \left(\frac{c}{a}\right)^2 \\ &= \frac{b^2}{a^2} + \frac{c^2}{a^2} \\ &= \frac{b^2 + c^2}{a^2} \end{aligned}$$



But by Pythagoras' Theorem, we also know that

$$a^2 = b^2 + c^2$$

Hence

$$\sin^2 \theta + \cos^2 \theta = \frac{a^2}{a^2} = 1$$

This result

$$\boxed{\sin^2 \theta + \cos^2 \theta = 1}$$

is a useful result. For example, when  $\theta = 45^\circ$ ,

$$\sin 45 = \cos 45 \quad (\text{as the two sides } c \text{ and } b \text{ are equal})$$

and hence,

$$\sin^2 45 + \sin^2 45 = 1$$

$$2 \sin^2 45 = 1$$

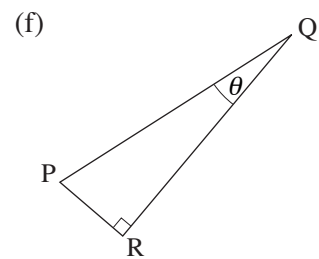
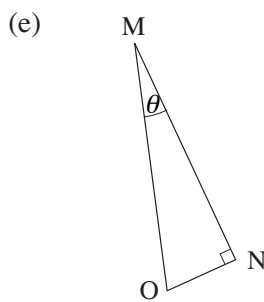
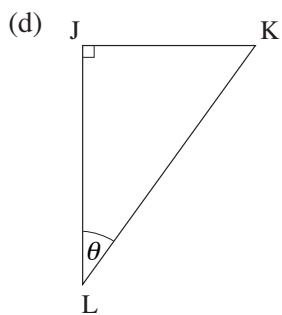
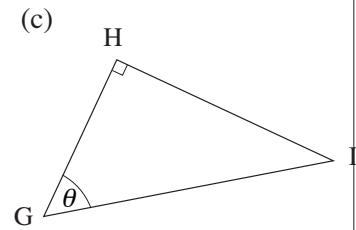
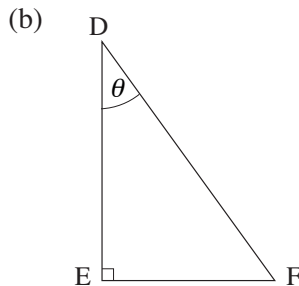
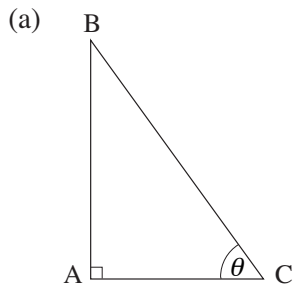
$$\sin^2 45 = \frac{1}{2}$$

$$\text{or} \quad \sin 45 = \cos 45 = \frac{1}{\sqrt{2}} \quad (\text{You can check this on your calculator.})$$

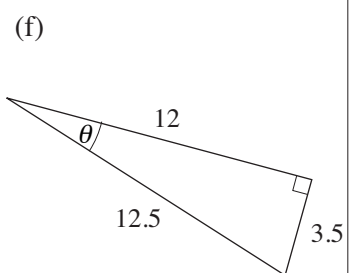
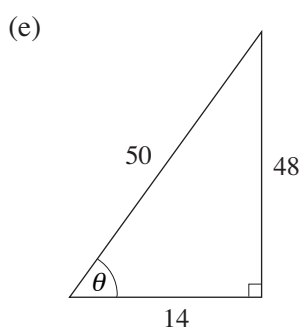
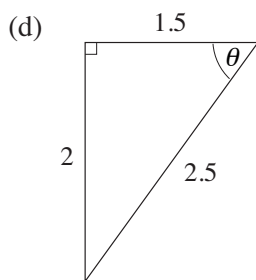
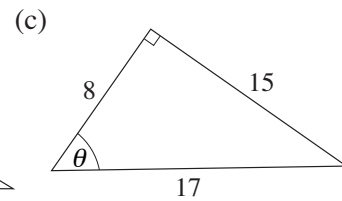
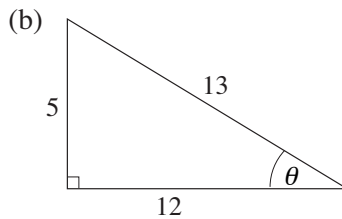
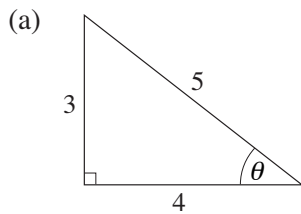


## Exercises

1. For each triangle, state which side is the hypotenuse, the adjacent and the opposite.



2. For each triangle, write  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$  as fractions.



3. Use a calculator to find the following. Give your answers correct to 3 decimal places.

(a)  $\sin 30^\circ$

(b)  $\tan 75^\circ$

(c)  $\tan 52.6^\circ$

(d)  $\cos 66^\circ$

(e)  $\tan 33^\circ$

(f)  $\tan 45^\circ$

(g)  $\tan 37^\circ$

(h)  $\sin 88.2^\circ$

(i)  $\cos 45^\circ$

(j)  $\cos 48^\circ$

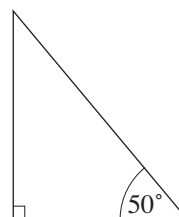
(k)  $\cos 46.7^\circ$

(l)  $\sin 45^\circ$

4. Use a calculator to find  $\theta$  in each case. Give your answers correct to 1 decimal place.

- |                           |                          |                          |
|---------------------------|--------------------------|--------------------------|
| (a) $\cos \theta = 0.5$   | (b) $\sin \theta = 1$    | (c) $\tan \theta = 0.45$ |
| (d) $\sin \theta = 0.821$ | (e) $\sin \theta = 0.75$ | (f) $\cos \theta = 0.92$ |
| (g) $\tan \theta = 1$     | (h) $\sin \theta = 0.5$  | (i) $\tan \theta = 2$    |
| (j) $\cos \theta = 0.14$  | (k) $\sin \theta = 0.26$ | (l) $\tan \theta = 5.25$ |

5. (a) Draw a right angled triangle with an angle of  $50^\circ$  as shown in the diagram, and measure the length of each side.



(b) Using

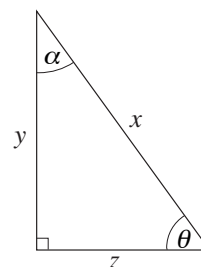
$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

and the lengths of the sides of your triangle, find  $\sin 50^\circ$ ,  $\cos 50^\circ$  and  $\tan 50^\circ$ .

- (c) Use your calculator to find  $\sin 50^\circ$ ,  $\cos 50^\circ$  and  $\tan 50^\circ$ .
- (d) Compare your results to (b) and (c).

6. For the triangle shown, write down expressions for:

- |                   |                   |
|-------------------|-------------------|
| (a) $\cos \theta$ | (b) $\sin \alpha$ |
| (c) $\tan \theta$ | (d) $\cos \alpha$ |
| (e) $\sin \theta$ | (f) $\tan \alpha$ |

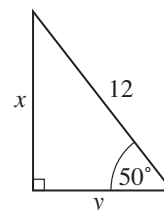


## 34.4 Finding Lengths in Right Angled Triangles

When one angle and the length of one side are known, it is possible to find the lengths of other sides in the same triangle, by using the sine, cosine or tangent formula.

For example,  $\sin 50^\circ = \frac{x}{12} \Rightarrow x = 12 \sin 50^\circ$

$$\cos 50^\circ = \frac{y}{12} \Rightarrow y = 12 \cos 50^\circ$$





### Worked Example 1

Find the length of the side marked  $x$  in the triangle shown.



#### Solution

In this triangle,      hypotenuse = 20 cm  
    opposite =  $x$  cm

Choose sine because it involves hypotenuse and opposite.

Using                               $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$

gives                               $\sin 70^\circ = \frac{x}{20}$

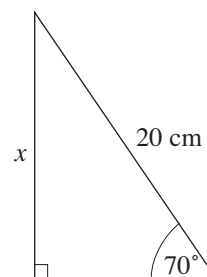
To obtain  $x$ , multiply both sides of this equation by 20, which gives

$$20 \sin 70^\circ = x$$

or

$$\begin{aligned} x &= 20 \sin 70^\circ \\ &= 18.8 \text{ cm} \quad (\text{to 1 decimal place}) \end{aligned}$$

This value is obtained using a calculator.



### Worked Example 2

Find the length of the side marked  $x$  in the triangle.



#### Solution

In this triangle,      opposite =  $x$   
    adjacent = 8 metres

Use tangent because it involves the opposite and adjacent.

Using                               $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

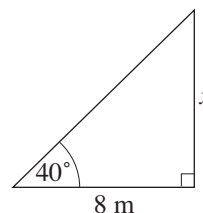
gives                               $\tan 40^\circ = \frac{x}{8}$

Multiplying both sides by 8 gives

$$8 \tan 40^\circ = x$$

or

$$\begin{aligned} x &= 8 \tan 40^\circ \\ &= 6.7 \text{ metres} \quad (\text{to 1 decimal place}) \end{aligned}$$





### Worked Example 3

Find the length marked  $x$  in the triangle.



### Solution

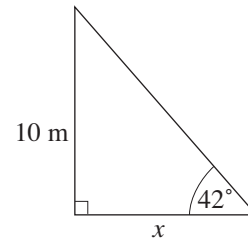
In this triangle,  $\tan 42^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{10}{x}$

Multiplying by  $x$  gives

$$x \tan 42^\circ = 10$$

and dividing by  $\tan 42^\circ$  gives

$$\begin{aligned} x &= \frac{10}{\tan 42^\circ} \\ &= \frac{10}{0.9004} \\ &= 11.1 \text{ metres} \quad (\text{to 1 decimal place}) \end{aligned}$$



### Worked Example 4

Find the length of the hypotenuse, marked  $x$ , in the triangle.



### Solution

In this triangle,  $\text{hypotenuse} = x$   
 $\text{opposite} = 10 \text{ cm}$

Use sine because it involves hypotenuse and opposite.

Using  $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$

gives  $\sin 28^\circ = \frac{10}{x}$

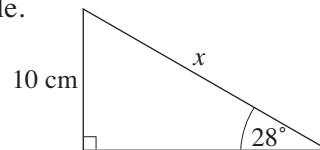
where  $x$  is the length of the hypotenuse.

Multiplying both sides by  $x$  gives

$$x \sin 28^\circ = 10,$$

then dividing both sides by  $\sin 28^\circ$  gives

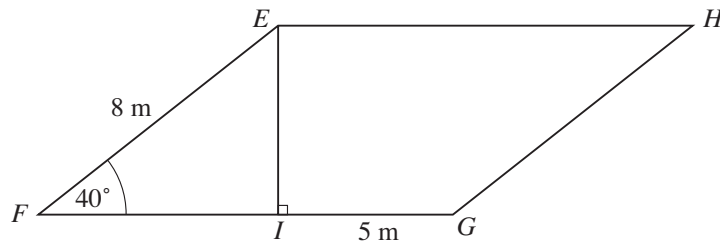
$$\begin{aligned} x &= \frac{10}{\sin 28^\circ} \\ &= 21.3 \text{ cm} \quad (\text{to 1 decimal place}) \end{aligned}$$







### Worked Example 5



The diagram above, **not drawn to scale**, represents one face of the roof of a house in the shape of a parallelogram  $EFGH$ . Angle  $EFI = 40^\circ$  and  $EF = 8$  m.  $EI$  represents a rafter placed perpendicular to  $FG$  such that  $IG = 5$  m.

Calculate, giving your answers to 3 significant figures,

- the length of  $FI$
- the length of  $EI$
- the area of  $EFGH$ .

(CXC)



### Solution

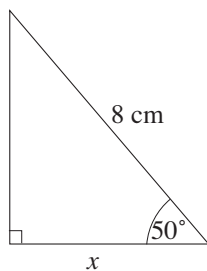
- $FI = 8 \cos 40 \approx 6.13$  m
- $EI = 8 \sin 40 \approx 5.14$  m
- Area of  $EFGH = FG \times EI$   
 $= (5 + 6.13) \times 5.14$   
 $= 11.13 \times 5.14$   
 $\approx 57.2 \text{ m}^2$



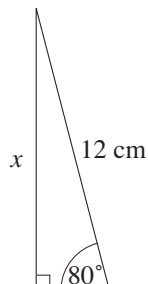
### Exercises

1. Find the length of the side marked  $x$  in each triangle.

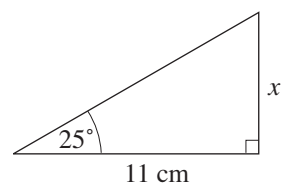
(a)

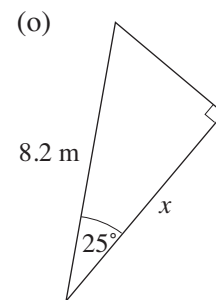
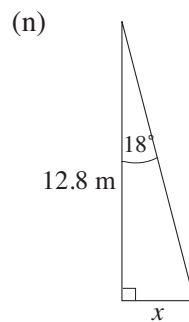
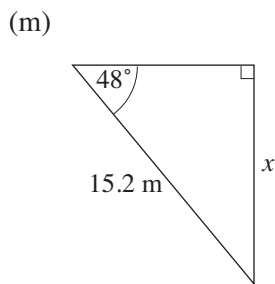
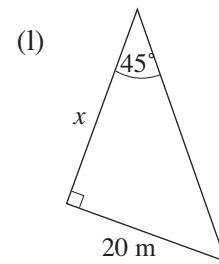
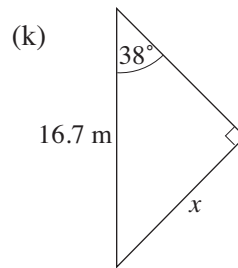
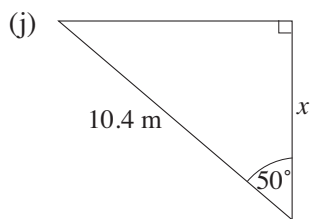
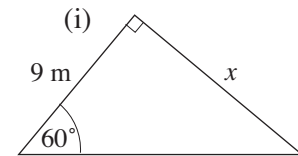
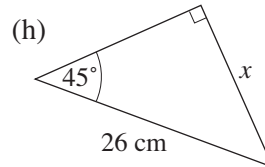
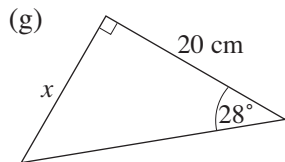
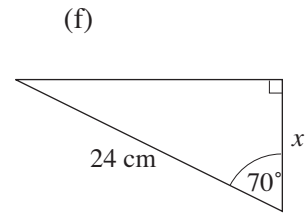
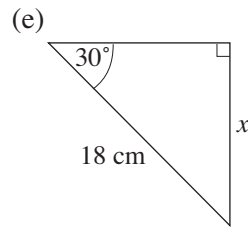
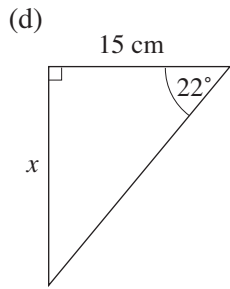


(b)

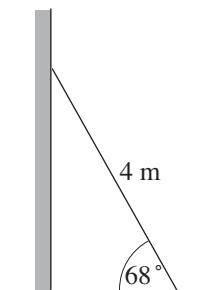


(c)



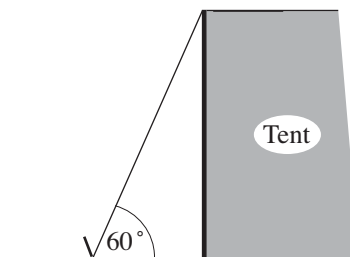


2. A ladder leans against a wall as shown in the diagram.

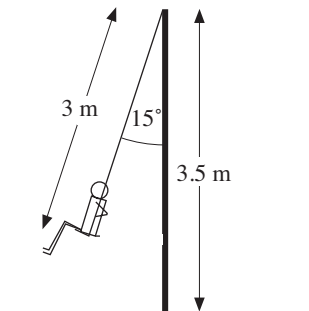


- (a) How far is the top of the ladder from the ground?
- (b) How far is the bottom of the ladder from the wall?

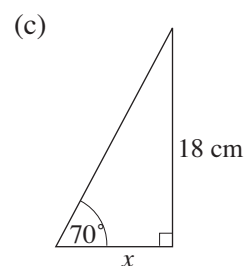
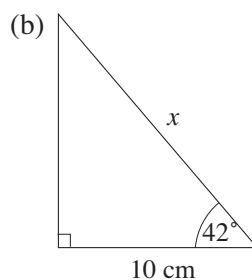
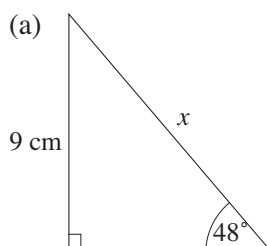
3. A guy rope is attached to a tent peg and the top of a tent pole so that the angle between the peg and the bottom of the pole is  $60^\circ$ .
- Find the height of the pole if the peg is 1 metre from the bottom of the pole.
  - If the length of the rope is 1.4 metres, find the height of the pole.
  - Find the distance of the peg from the base of the pole if the length of the guy rope is 2 metres.

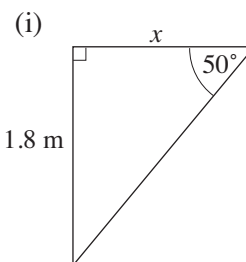
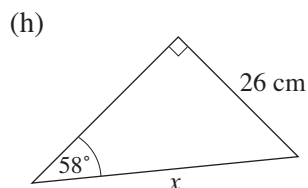
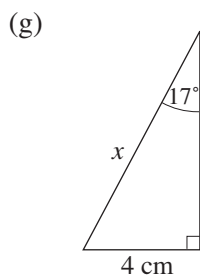
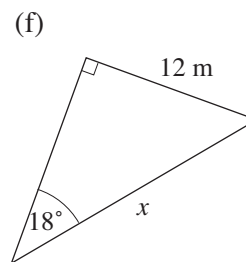
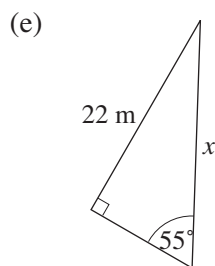
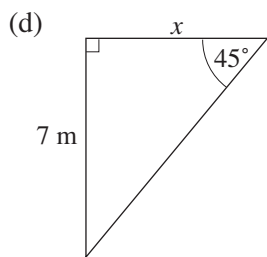


4. A child is on a swing in a park. The highest position that she reaches is as shown.
- Find the height of the swing seat above the ground in this position.

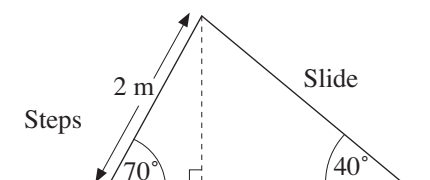


5. A laser beam shines on the side of a building. The side of the building is 500 metres from the source of the beam, which is at an angle of  $16^\circ$  above the horizontal. Find the height of the point where the beam hits the building.
6. A ship sails 400 km on a bearing of  $075^\circ$ .
- How far east has the ship sailed?
  - How far north has the ship sailed?
7. An aeroplane flies 120 km on a bearing of  $210^\circ$ .
- How far south has the aeroplane flown?
  - How far west has the aeroplane flown?
8. A kite has a string of length 60 metres. On a windy day all the string is let out and makes an angle of between  $20^\circ$  and  $36^\circ$  with the ground. Find the minimum and maximum heights of the kite.
9. Find the length of the side marked  $x$  in each triangle.

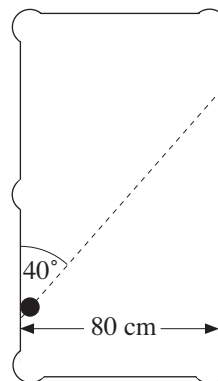




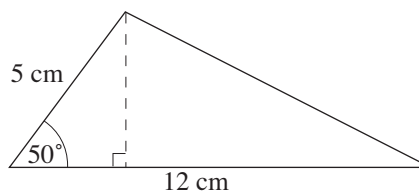
10. The diagram shows a slide in a playpark.  
 (a) Find the height of the top of the slide.  
 (b) Find the length of the slide.



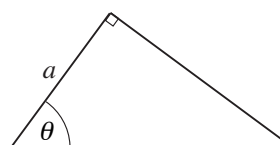
11. A snooker ball rests against the side cushion of a snooker table. It is hit so that it moves at  $40^\circ$  to the side of the table. How far does the ball travel before it hits the cushion on the other side of the table?



12. (a) Find the length of the dotted line and the area of this triangle.

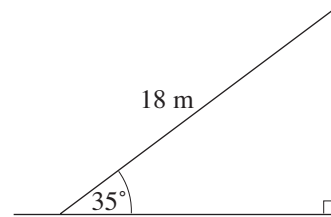


- (b) Find the height of this triangle and then find a formula for its area in terms of  $a$  and  $\theta$ .



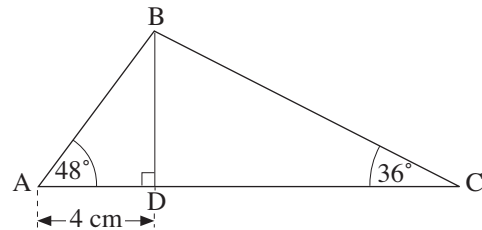
13. A wire 18 metres long runs from the top of a pole to the ground, as shown in the diagram. The wire makes an angle of  $35^\circ$  with the ground.

Calculate the height of the pole. Give your answer to a reasonable degree of accuracy.



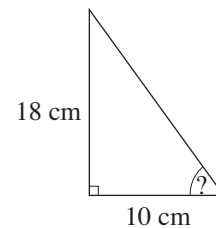
14. In the figure shown, calculate

- (a) the length of BD.  
(b) the length of BC.



## 34.5 Finding Angles in Right Angled Triangles

If the lengths of any two sides of a right angled triangle are known, then sine, cosine and tangent can be used to find the angles of the triangle.



### Worked Example 1

Find the angle marked  $\theta$  in the triangle shown.



### Solution

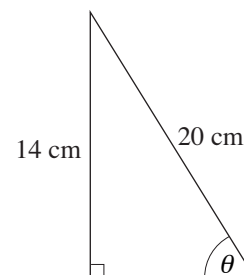
In this triangle,      hypotenuse = 20 cm  
   opposite = 14 cm

Using                       $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$

gives                       $\sin \theta = \frac{14}{20}$   
   = 0.7

So                               $\theta = \sin^{-1} (0.7)$

and using the **(SHIFT)** and **(SIN)** buttons on a calculator gives  $\theta = 44.4^\circ$  (to 1 d.p.)





## Worked Example 2

Find the angle marked  $\theta$  in the triangle shown.



### Solution

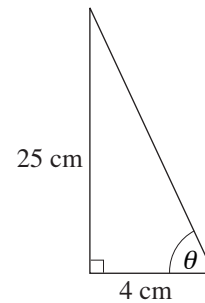
In this triangle,      opposite = 25 cm  
    adjacent = 4 cm

Using                       $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

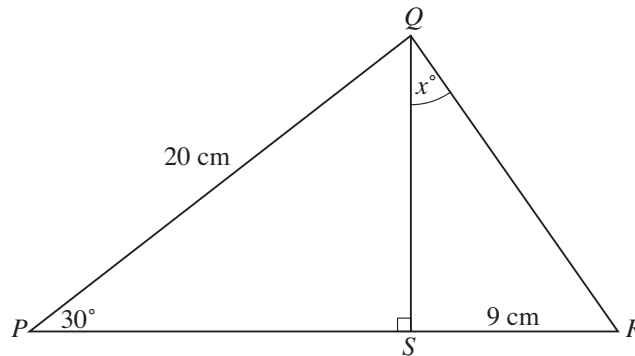
gives                       $\tan \theta = \frac{25}{4}$   
    = 6.25

So                               $\theta = \tan^{-1} (6.25)$

and using the **(SHIFT)** and **(TAN)** buttons on a calculator gives  $\theta = 80.9^\circ$  (to 1 d.p.)



## Worked Example 3



The diagram above shows triangle  $PQR$ , **not drawn to scale**.

$PQ = 20$  cm,  $\angle QPR = 30^\circ$ ,  $QS$  is perpendicular to  $PR$ ,  $SR = 9$  cm, and  $\angle SQR = x^\circ$ .

Calculate

- the length of  $QS$
- the size of angle  $x$  to the nearest degree.



### Solution

- $QS = 20 \sin 30^\circ$   
           = 10 cm

$$(b) \quad \tan x = \frac{9}{10}$$

$$= 0.9$$

$$x = \tan^{-1}(0.9)$$

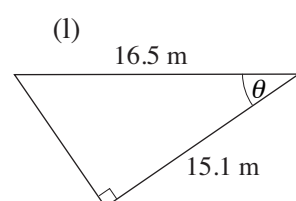
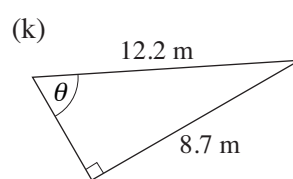
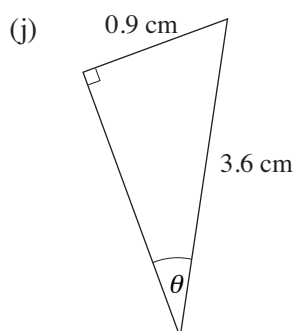
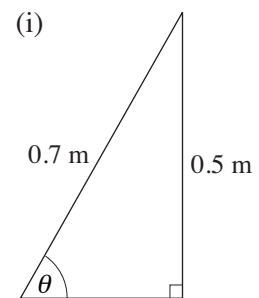
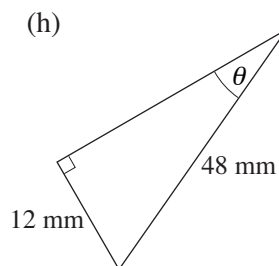
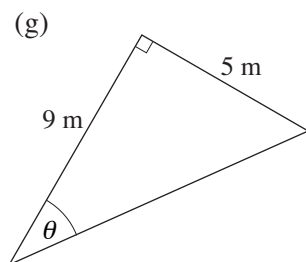
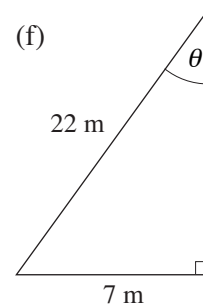
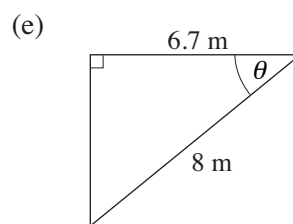
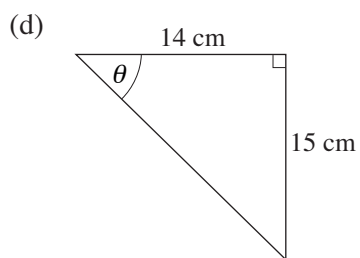
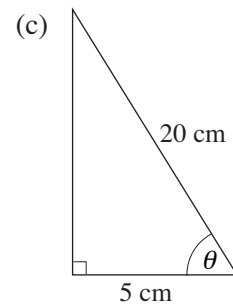
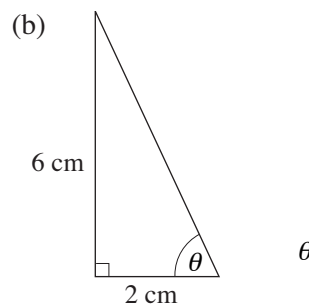
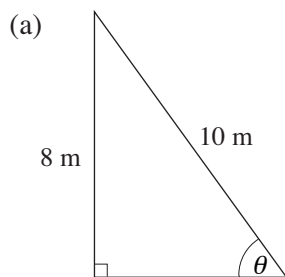
and using the **SHIFT** and **TAN** buttons on a calculator gives

$$x \approx 41.987^\circ \approx 42^\circ \text{ to the nearest degree.}$$

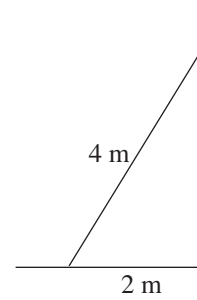


## Exercises

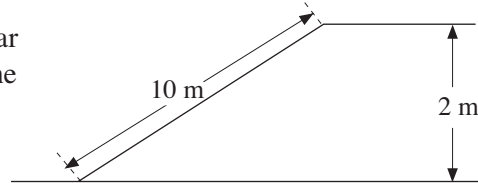
1. Find the angle  $\theta$  of:



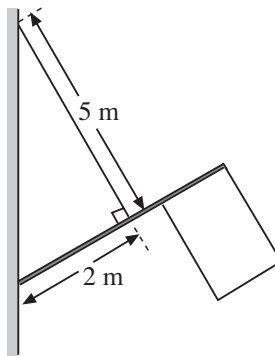
2. A ladder leans against a wall. The length of the ladder is 4 metres and the base is 2 metres from the wall. Find the angle between the ladder and the ground.



3. As cars drive up a ramp at a multi-storey car park, they go up 2 metres. The length of the ramp is 10 metres. Find the angle between the ramp and the horizontal.



4. A flag pole is fixed to a wall and supported by a rope, as shown.

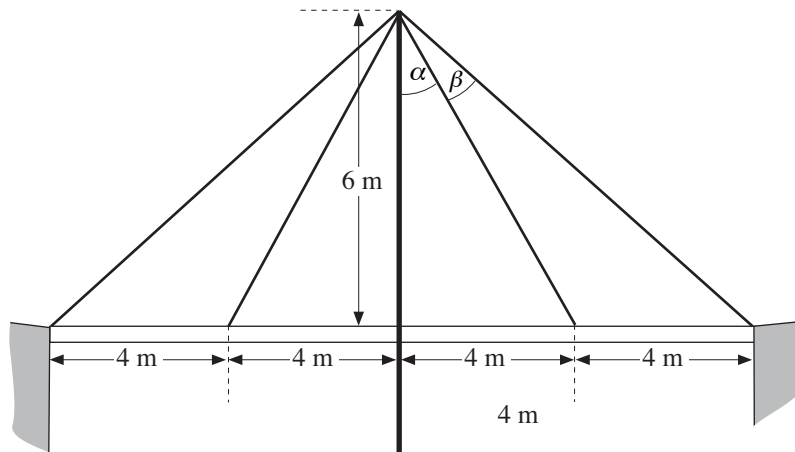


Find the angle between

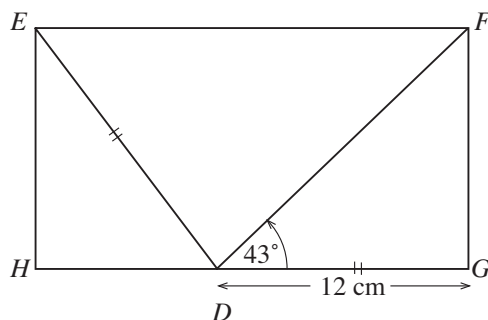
- (a) the rope and the wall
- (b) the pole and the wall.
5. The mast on a yacht is supported by a number of wire ropes. One, which has a length of 15 metres, goes from the top of the mast at a height of 10 metres, to the front of the boat.
- (a) Find the angle between the wire rope and the mast.
- (b) Find the distance between the base of the mast and the front of the boat.
6. A soldier runs 500 metres east and then 600 metres north. If he had run directly from his starting point to his final position, what bearing should he have run on?
7. A ship is 50 km south and 70 km west of the port that it is heading for. What bearing should it sail on to reach the port?



8. The diagram shows a simple bridge, which is supported by four steel cables.
- Find the angles at  $\alpha$  and  $\beta$ .
  - Find the length of each cable.

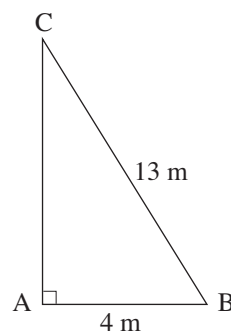


9. In the diagram below, **not drawn to scale**,  $EFGH$  is a rectangle. The point  $D$  on  $HG$  is such that  $ED = DG = 12$  cm and  $\hat{GDF} = 43^\circ$ .



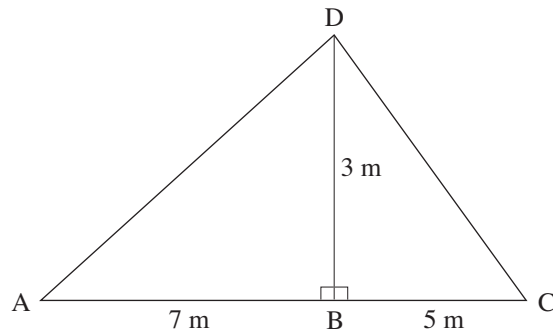
Calculate correct to one decimal place

- the length of  $GF$
  - the length of  $HD$
  - the size of the angle  $HDE$ . (CXC)
10.  $ABC$  is a right angled triangle.  $AB$  is of length 4 metres and  $BC$  is of length 13 metres.
- Calculate the length of  $AC$ .
  - Calculate the size of angle  $ABC$ .



11. The diagram shows a roofing frame ABCD.

$AB = 7 \text{ m}$ ,  $BC = 5 \text{ m}$ , angle  $ABD = \text{angle } DBC = 90^\circ$



- (a) Calculate the length of AD.
- (b) Calculate the size of angle DCB.