Simultaneous equations

Simultaneous equations are among the most exciting type of equations that you can learn in mathematics. That's especially because they are very adaptable and applicable to practical situations. Simultaneous equations may be solved by

(a) Matrix Methods
(b) Graphically
(c) Algebraic methods

But first, why are they called simultaneous equations?

Consider the following equation $5x+2=17$, solving this equation gives

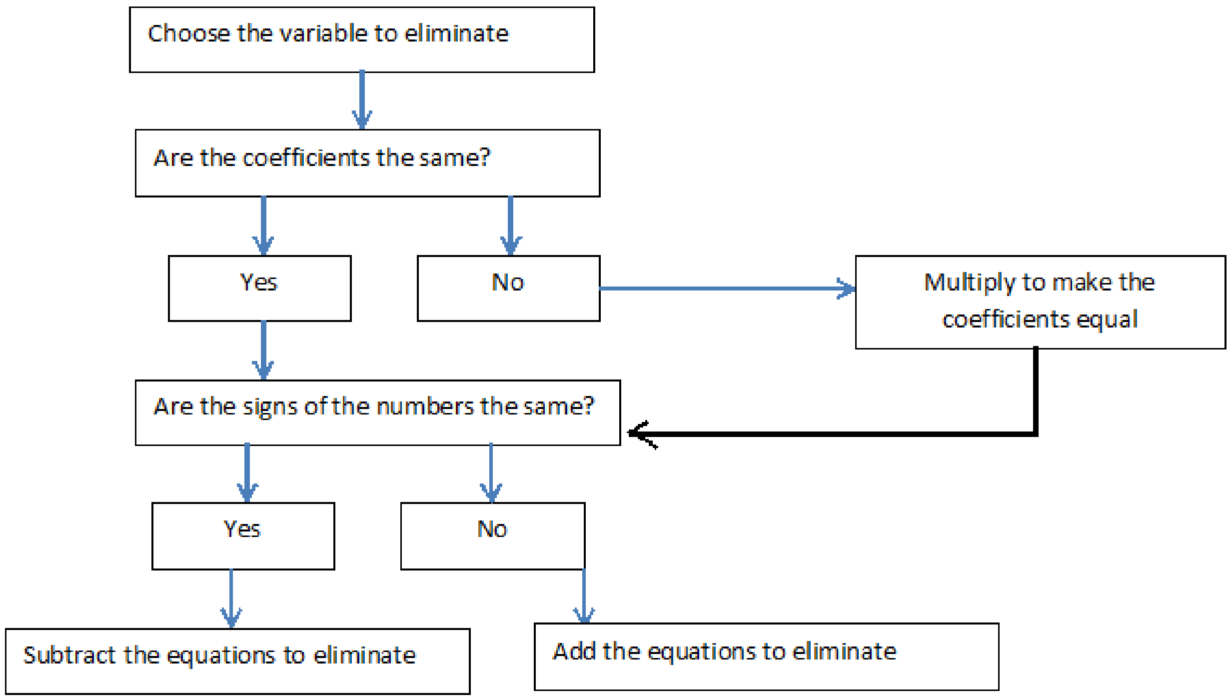
$$\begin{aligned}5x+2&=17\\5x&=17-2\\5x&=15\\x&=\frac{15}{5}=3\end{aligned}$$

We say $x=3$ is a unique solution because it is the only number that can make the equation or statement true so means $5x+2=17$ gives $\begin{aligned}5(3)+2&=17\\15+2&=17\\17&=17\end{aligned}$

Now consider this equation $a+b=10$, there are infinitely many solutions to this equation and these solutions would not be unique. But suppose we combine this equation with another one similar to it, such as $a-b=4$ and attempt to solve them together as one system $\begin{aligned}a+b&=10\\a-b&=4\end{aligned}$. We call them simultaneous equations because we are trying to solve them together, at the same time, in tandem or simultaneously.

We will be concentrating on algebraic methods and especially on the elimination method which means literally to get rid of one of the variables

The Elimination Method in Detail



Example 1

When I add two numbers I get 12 and when I subtract them I get 2

Solution

Let a be the larger number and b the smaller number then we can represent our equations as

$a+b=12$

$a-b=2$

Let's choose to eliminate the "***a***" first. Notice that the coefficients are the same, [both are 1], also notice that their signs are different as in one is positive and the other negative so in this case we add the equations

$a+b=12$

$a-b=2$

$(a+a)+(b+-b)=(12+2)$;

$2a+0=14$

$a=\frac{14}{2}=7$

By substituting our 7 into one of the equations we can find the value of the other letter ***b***.

$a+b=12$

$7+b=12$

$b=12-7$

$b=5$

Example 2

Pat and Jane stopped by a fruit stand, pat bought two oranges and three mangoes for $82. Jane bought four similar oranges and two similar mangoes for $108. What is the cost of a mango and the cost of an orange?

Let $x =$ the cost of an orange and $y =$ the cost of a mango. Then we can represent the equations as

$$\begin{aligned} 2x + 3y &= 82 \\ 4x + 2y &= 108 \end{aligned}$$

Notice here that the coefficients for the variables are not equal. For $\boldsymbol{y}$ we have 3 and 2 and for $\boldsymbol{x}$ we have 2 and 4. If we choose to eliminate the $\boldsymbol{x}$, we could multiply $2x + 3y = 82$ by 2 to give

$$\begin{aligned} &2(2x + 3y = 82) \\ &4x + 6y = 164 \end{aligned}$$

notice that the signs are the same; that is both are positive, in this case we subtract the equations and that would mean that the coefficients of the $\boldsymbol{x}$ are equal so we now have

$$\begin{aligned} 4x + 6y &= 164 \\ 4x + 2y &= 108 \end{aligned}$$

and solving them we get

$$\begin{aligned} &4x + 6y = 164 \\ &-(4x + 2y = 108) \\ &(4x - 4x) + (6y - 2y) = (164 - 108) \\ &4y = 56 \\ &y = \$14 \end{aligned}$$

If we chose to eliminate the y we would have to use a different multiplication

$$\begin{aligned} &2x + 3y = 82 \\ &4x + 2y = 108 \\ &2(2x + 3y = 82) \Rightarrow 4x + 6y = 164 \\ &3(4x + 2y = 108) \Rightarrow 12x + 6y = 324 \\ &4x + 6y = 164 \\ &12x + 6y = 324 \end{aligned}$$

Subtracting the equations we get

$$\begin{aligned} &12x + 6y = 324 \\ &-(4x + 6y = 164) \\ &(12x - 4x) + (6y - 6y) = (324 - 164) \\ &8x = 160 \\ &x = \$20 \end{aligned}$$

So an orange costs $14 and a mango costs $20

Example 3

Denise sells 300 tickets for a concert. Some tickets are sold to adults for $5 and some for $4 to children. If she collects $1444 in ticket sales how many tickets were sold to adults and how many to children

Let $x =$ the number of $5 tickets and

Let $y =$ the number of $4 tickets

The system of equations can be represented as

$$\begin{aligned} x + y &= 300 \\ 5x + 4y &= 1444 \end{aligned}$$

If we choose to eliminate the y then we multiply $x + y = 300$ by 4 to give

$$4(x + y = 300) \Rightarrow 4x + 4y = 1200$$

And solving we get

$$\begin{aligned} &4(x + y = 300) \Rightarrow 4x + 4y = 1200 \\ &5x + 4y = 1444 \\ &-(4x + 4y = 1200) \\ &(5x - 4x) + (4y - 4y) = (1444 - 1200) \\ &x = 244 \end{aligned}$$

By substitution we can obtain the other solution which is

$$\begin{aligned} &x + y = 300 \\ &x = 244 \\ &y = 300 - 244 = 56 \end{aligned}$$

Example 4

John and David have \$14 together. If John's money is doubled and David's money tripled they will have \$34 together. How much money does each boy have?

Let $x =$ John's money

Let $y =$ David's money

We can represent the equations as

$$x + y = 14$$
$$2x + 3y = 34$$

If we choose to eliminate the y then we multiply $x + y = 14$ by 3 to give

$$3(x + y = 14) = 3x + 3y = 42$$

Solving together we get

$$3x + 3y = 42$$
$$-(2x + 3y = 34)$$
$$(3x - 2x) + (3y - 3y) = (42 - 34)$$
$$x = 8$$

And substituting we get

$$x + y = 14$$
$$x = 8$$
$$y = 14 - 8 = 6$$

Practice questions

1. $$3x + 2y = 7$$
   $$5x - 2y = 1$$

2. $$7a - 3b = 26$$
   $$a + 2b = 11$$

3. $$2m + 2n = 16$$
   $$5m + n = 21$$

4. $$2x + 3y = 23$$
   $$x + y = 4$$

5. $$3x + 3y = 15$$
   $$3x - 5y = -41$$

6. Four mangoes and two pears cost $24 while two mangoes and three pears cost $16.
   a. Write a pair of simultaneous equations in $\boldsymbol{x}$ and $\boldsymbol{y}$ to represent the information given above
   b. State clearly what the $\boldsymbol{x}$ and $\boldsymbol{y}$ represent
   c. By solving the pair of equations determine the cost of both fruits

7. 5 lollipops and 12 toffees have a mass of 61 grams , also 10 lollipops and 13 toffees have a mass of 89 grams
   a. Write a pair of simultaneous equations in $\boldsymbol{x}$ and $\boldsymbol{y}$ to represent the information given above
   b. Calculate the cost of each item

8. One packet of biscuits costs $x and one cup of ice cream costs $y. One packet of biscuits and two cups of ice-cream costs $8, while three packets of biscuits and one cup of ice cream costs $9

a. Write a pair of simultaneous equations in x and y to represent the information given above
b. By solving the pair of equations determine the cost of both items

9. A stadium has two sections, A and B, tickets for section A costs $\$a$ each and tickets for section B costs $\$b$ each. Johanna paid $105 for 5 section A tickets and 3 section B tickets. Raiyah paid $63 for 4 section A tickets and 1 section B ticket
   a. Write the equations in **a** and b to represent the information
   b. Calculate the values of a and b

10. The perimeter of a rectangle is 14m. If the length is doubled and the width is halved the new perimeter is 22m. By using a pair of simultaneous equations determine the length and width of the rectangle.

11. A restaurant bill of $3200 is paid with $500 bills and $100 bills. If 16 bills were used determine how many $100 and how many $500 bills were used.