STRAND G: Algebra

Unit 21 *Formulae*

Student Text

Contents

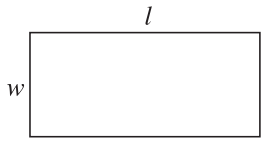
21 Formulae

21.1 Using Formulae

In formulae, letters are used to represent numbers. For example, the formula

$$A = lw$$

can be used to find the area of a rectangle. Here A is the area, l the length and w the width. In this formula, lw means $l \times w$. Formulae are usually written in this way, without multiplication signs.



The perimeter of the rectangle would be given by the formula

$$P = 2l + 2w$$

Here again there are no multiplication signs, and $2l$ means $2 \times l$ and $2w$ means $2 \times w$.

Worked Example 1

The perimeter of a rectangle can be found using the formula

$$P = 2l + 2w$$

Find the perimeter if $l = 8$ and $w = 4$.

Solution

The letters l and w should be replaced by the numbers 8 and 4.

This gives

$$\begin{aligned} P &= 2 \times 8 + 2 \times 4 \\ &= 16 + 8 \\ &= 24 \end{aligned}$$

Worked Example 2

The final speed of a car is v and can be calculated using the formula

$$v = u + at$$

where u is the initial speed, a is the acceleration and t is the time taken.

Find v if the acceleration is $2\ \text{m s}^{-1}$, the time taken is 10 seconds and the initial speed is $4\ \text{m s}^{-1}$.

Solution

The acceleration is $2\ \text{m s}^{-1}$ so $a = 2$. The initial speed is $4\ \text{m s}^{-1}$ so $u = 4$.

The time taken is $10\ \text{s}$ so $t = 10$.

Using the formula

$$v = u + at$$

gives

$$\begin{aligned} v &= 4 + 2 \times 10 \\ &= 4 + 20 \\ &= 24\ \text{m s}^{-1} \end{aligned}$$

Exercises

1. The area of a rectangle is found using the formula $A = lw$ and the perimeter using $P = 2l + 2w$. Find the area and perimeter if:

   (a) $l = 4$ and $w = 2$ (b) $l = 10$ and $w = 3$

   (c) $l = 11$ and $w = 2$ (d) $l = 5$ and $w = 4$

2. The formula $v = u + at$ is used to find the final speed.

   Find v if`:

   (a) $u = 6$, $a = 2$ and $t = 5$ (b) $u = 0$, $a = 4$ and $t = 3$

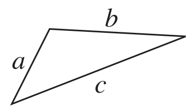
   (c) $u = 3$, $a = 1$ and $t = 12$ (d) $u = 12$, $a = 2$ and $t = 4$

3. Use the formula $F = ma$ to find F if:

   (a) $m = 10$ and $a = 3$ (b) $m = 200$ and $a = 2$

4. The perimeter of a triangle is found using the formula

$$P = a + b + c$$



   Find P if:

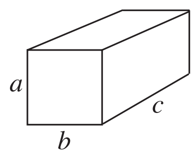
   (a) $a = 10$, $b = 12$ and $c = 8$

   (b) $a = 3$, $b = 4$ and $c = 5$

   (c) $a = 6$, $b = 4$ and $c = 7$

5. The volume of a box is given by the formula

$$V = abc$$



   Find V if:

   (a) $a = 2$, $b = 3$ and $c = 10$

   (b) $a = 7$, $b = 5$ and $c = 3$

   (c) $a = 4$, $b = 4$ and $c = 9$

6. Find the value of Q for each formula using the values given.

   (a) $Q = 3x + 7y$
   $x = 4$ and $y = 2$

   (b) $Q = x^2 + y$
   $x = 3$ and $y = 5$

   (c) $Q = xy + 4$
   $x = 3$ and $y = 5$

   (d) $Q = 5x - 2y$
   $x = 10$ and $y = 2$

(e) $Q = xy - 2$

$x = 10$ and $y = 2$

(f) $Q = \dfrac{x}{y}$

$x = 24$ and $y = 2$

(g) $Q = \dfrac{x + 4}{y}$

$x = 8$ and $y = 3$

(h) $Q = \dfrac{4x + 2}{y}$

$x = 5$ and $y = 11$

(i) $Q = 3x + 2y + z$

$x = 4$, $y = 2$ and $z = 10$

(j) $Q = xy + yz$

$x = 2$, $y = 5$ and $z = 8$

(k) $Q = xyz$

$x = 2$, $y = 5$ and $z = 3$

(l) $Q = xy + 4z$

$x = 8$, $y = 3$ and $z = 4$

(m) $Q = \dfrac{x + y}{z}$

$x = 8$, $y = 10$ and $z = 3$

(n) $Q = \dfrac{x}{y + z}$

$x = 50$, $y = 2$ and $z = 3$

7. This formula is used to work out Aleale's pay:

Pay = \$20 + Number of hours worked × Rate of pay

Her rate of pay is \$20 plus \$9 per hour.

Work out her pay for 40 hours.

8. A rectangle has a length of *a* cm and a width of *b* cm.

The perimeter of a rectangle is given by the formula $p = 2(a + b)$.

Calculate the perimeter of a rectangle when $a = 4.5$ and $b = 4.2$.

21.2 Construct and Use Simple Formulae

A *formula* describes how one quantity relates to one or more other quantities. For example, a formula for the area of a rectangle describes how to find the area, given the length and width of the rectangle.

The perimeter of the rectangle would be given by the formula

$$P = 2l + 2w$$

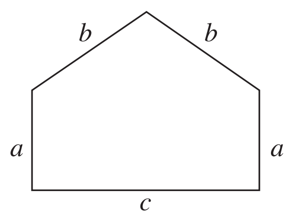
Here again there are no multiplication signs and $2l$ means $2 \times l$ and $2w$ means $2 \times w$.

Worked Example 1

(a) Write down a formula for the perimeter of the shape shown.

(b) Find the perimeter if

$a = 2$ cm, $b = 3$ cm and $c = 5$ cm



Solution

(a) The perimeter is found by adding together the lengths of all the sides, so the formula will be

$$P = a + b + b + a + c$$

but as a and b are both added in twice, this can be simplified to

$$P = 2a + 2b + c$$

(b) If $a = 2$ cm, $b = 3$ cm and $c = 5$ cm,

$$\begin{aligned} P &= 2 \times 2 + 2 \times 3 + 5 \\ &= 4 + 6 + 5 \\ &= 15 \text{ cm} \end{aligned}$$

Worked Example 2

When repairing air conditioning systems, an emergency engineer charges a basic fee of J$3000 plus J$1200 per hour.

Find a formula for calculating the engineer's charge.

Solution

Let C = charge and n = number of hours.

The charge is made up of

a fixed J$3000 and J$1200 $\times$ the number of hours or J1200n$.

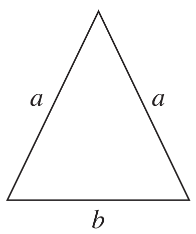
So the total charge in J$ is given by

$$C = 3000 + 1200n$$

Exercises

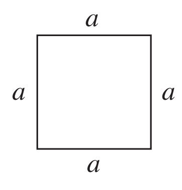
1. Find a formula for the perimeter of each shape, and find the perimeter for the specified values.

(a)



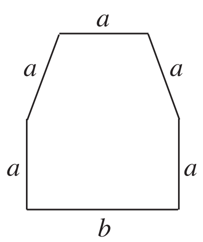
$a = 6$ cm, $b = 4$ cm

(b)



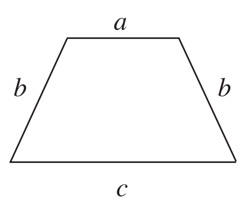
$a = 5$ cm

(c)



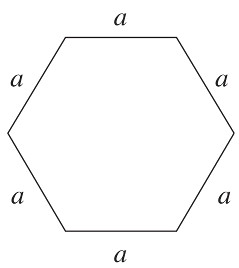
$a = 6$ cm, $b = 10$ cm

(d)



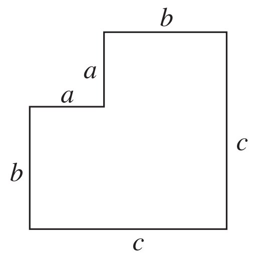
$a = 5$ cm, $b = 6$ cm, $c = 10$ cm

(e)



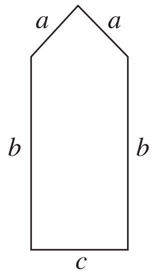
$a = 10$ cm

(f)



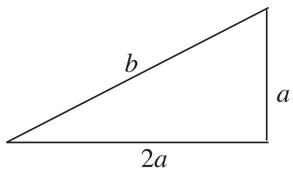
$a = 4$ cm, $b = 5$ cm, $c = 9$ cm

(g)



$a = 60$ cm, $b = 160$ cm,
$c = 80$ cm

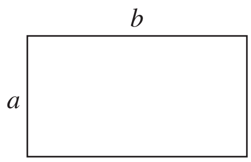
(h)



$a = 4$ cm, $b = 9$ cm

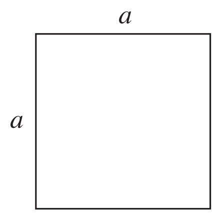
2. Find a formula for the area of each of the shapes below and find the area for the values given.

(a)



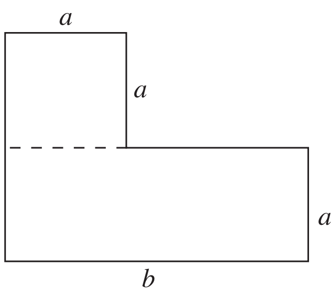
$a = 6$ cm, $b = 10$ cm

(b)



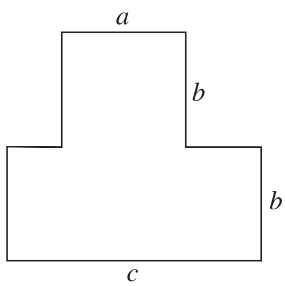
$a = 3$ cm

(c)



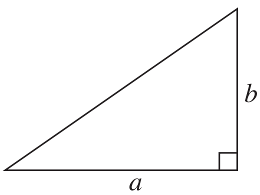
$a = 2$ cm, $b = 8$ cm

(d)



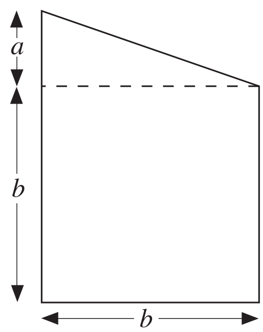
$a = 3$ cm, $b = 4$ cm, $c = 9$ cm

(e)



$a = 4$ cm, $b = 5$ cm

(f)



$a = 50$ cm, $b = 200$ cm

3. Three consecutive numbers are to be added together.

   (a) If x is the smallest number, what are the other two numbers in terms of x?

   (b) Write down a formula for the total, T, of the three numbers in terms of x, using your answer to (a).

4. (a) Write down a formula to find the mean, M, of the two numbers x and y.

   (b) Write down a formula to find the mean, M, of the five numbers $p, q, r, s,$ and t.

5. Tickets for a school concert are sold at $6 for adults and $4 for children.

   (a) If p adults and q children buy tickets, write a formula for the total value, T, of the ticket sales.

   (b) Find the total value of the ticket sales if $p = 50$ and $q = 20$.

6. A rectangle is 3 cm longer than it is wide.

   If x is the width, write down a formula for:

   (a) the perimeter, P;

   (b) the area, A, of the rectangle.

7. Rachel is one year older than Bradley. Carla is three years younger than Bradley

   If Bradley is x years old, write down expressions for:

   (a) Rachel's age;

   (b) Carla's age;

   (c) the sum of all three children's ages.

8. A window cleaner charges a fee of J$800 for visiting a house and J$400 for every window that he cleans.

   (a) Write down a formula for finding the total cost C, in J$, when n windows are cleaned.

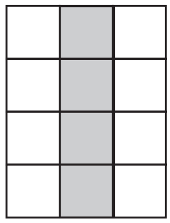
   (b) Find C if $n = 8$.

9. A taxi driver charges a fee of $5, plus $4 for every km that the taxi travels.

   (a) Find a formula for the cost C of a journey that covers x km.

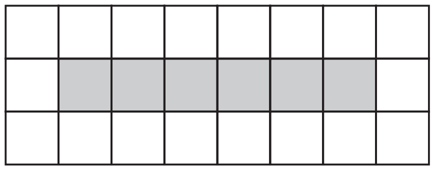
   (b) Find C if $x = 3$.

10. A gardener builds paths using paving slabs laid out in a pattern as shown, with white slabs on each side of a row of red slabs.



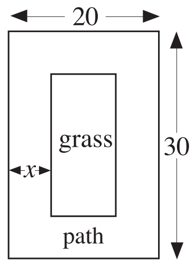
(a) If n red slabs are used, how many white slabs are needed?

(b) Another gardener puts a white slab at each end of the path as shown below.



If n red slabs are used, how many white slabs are needed?

11. A path of width x is laid around a rectangular lawn as shown.



(a) Find an expression for the perimeter of the grass.

(b) Find an expression for the area of the grass.

12. Juice drinks cost J$27 each.
Write down a formula for the cost, J$$C$, of n drinks.

13. (a) Gas costs J$80 per litre.
Write down a formula for the cost, J$$C$, of l litres of gas.

(b) Gas costs J$ x per litre.
Write down a formula for the cost, J$$C$, of l litres of gas.

14. Write down an expression for the TOTAL cost, in dollars, of 8 metres of fabric at x dollars per metre and y reels of thread at 2 dollars per reel.

15. Mr James works a basic week of 40 hours at a rate of $16 an hour. His overtime rate is $4 per hour MORE than his basic rate.
Calculate

(i) his TOTAL wage for a basic week.

(ii) his wage for a week in which he worked 47 hours.

21.3 Revision of Negative Numbers

Before starting the next section on formulae it is useful to revise how to work with negative numbers.

Note

When multiplying or dividing two numbers, if they have the *same* sign the result will be *positive*, but if they have *different* signs the result will be *negative*.

Worked Example 1

Find

(a) $(-3)\times(-7)$ (b) $(-24)\div 3$

(c) $(-40)\div(-5)$ (d) $(-6)\times 7$

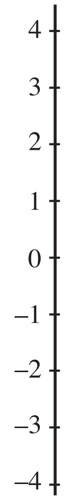
Solution

(a) $(-3)\times(-7) = 21$ (b) $(-24)\div 3 = -8$

(c) $(-40)\div(-5) = 8$ (d) $(-6)\times 7 = -42$

Note

When adding or subtracting it can be helpful to use a *number line*, remembering to move up when adding and down when subtracting a positive number. When adding a negative number, move down and when subtracting a negative number, move up.



Number line

Worked Example 2

Find

(a) $4 - 10$ (b) $-6 + 8$

(c) $-4 - 5$ (d) $-6 + (-7)$

(e) $7 - (-4)$

Solution

(a) $4 - 10 = -6$ (b) $-6 + 8 = +2$

(c) $-4 - 5 = -9$ (d) $-6 + (-7) = -6 - 7$
$= -13$

(e) $7 - (-4) = 7 + 4$
$= 11$

Exercises

1. (a) $6 - 8 =$ (b) $-8 + 12 =$ (c) $-5 + 2 =$

(d) $-6 - 2 =$ (e) $(-8)\times(-3) =$ (f) $(-9)\times(-6) =$

(g) $(-24)\div(-3) =$ (h) $16\div(-2) =$ (i) $(-81)\div(-3) =$

(j) $-16 + 24 =$ (k) $-8 - 5 =$ (l) $(-5)\times 7 =$

(m) $3\times(-8) =$ (n) $-1 - 10 =$ (o) $-10 + 5 =$

(p) $9 + (-6) =$ (q) $4 - (-7) =$ (r) $-1 - (-4) =$

(s) $-1 + (-7) =$ (t) $-4 + (-2) =$ (u) $-6 - (-5) =$

2. (a) $(-1)^2$ (b) $(-4)^2$ (c) $(-4)^2 + (-3)^2$

3. In London, the temperature at midday was $5\ °C$.

   At midnight the temperature had fallen by $8\ °C$.

   What was the temperature at midnight?

4. The temperature was recorded inside and outside a house in New York.

Inside temperature	Outside temperature
16 °C	−8 °C

   How many degrees warmer was it inside the house than outside?

Challenge!

You open a book. Two pages face you. If the product of the two page numbers is 3192, what are the two page numbers?

21.4 Substitution into Formulae

The process of replacing the letters in a formula is known as *substitution*.

Worked Example 1

The length of a metal rod is l cm. The length changes with temperature and can be found by the formula

$$l = 40 + 0.02T$$

where T is the temperature.

Find the length of the rod when

(a) $T = 50\ °C$ and (b) $T = -10\ °C$

Solution

(a) Using $T = 50$ gives

$$\begin{aligned} l &= 40 + 50 \times 0.02 \\ &= 40 + 1 \\ &= 41 \text{ cm} \end{aligned}$$

(b) Using $T = -10$ gives

$$\begin{aligned} l &= 40 + (-10) \times 0.02 \\ &= 40 + (-0.2) \\ &= 40 - 0.2 \\ &= 39.8 \text{ cm} \end{aligned}$$

Worked Example 2

The profit in $ made by a salesman when he sells n books is calculated by the formula

$$P = 4n - 50$$

Find the profit if he makes

(a) 30 sales (b) 9 sales

Solution

(a) Here $n = 30$ so the formula gives

$$\begin{aligned} P &= 4 \times 30 - 50 \\ &= 120 - 50 \\ &= 70 \end{aligned}$$

So the salesman's profit after 30 sales is $70.

(b) Here $n = 9$ so the formula gives

$$\begin{aligned} P &= 4 \times 9 - 50 \\ &= 36 - 50 \\ &= -14 \end{aligned}$$

He makes a loss of $14 if only 9 sales are made.

Worked Example 3

If $x * y$ is defined by

$$x * y = x^2 - 4y^2$$

what is the value of $x * y$ when

(a) $x = 4$, $y = -2$

(b) $x = -2$, $y = 4$?

Solution

(a)
$$\begin{aligned} 4 * (-2) &= 4^2 - 4(-2)^2 \\ &= 16 - 4 \times 4 \\ &= 16 - 16 \\ &= 0 \end{aligned}$$

(b)
$$\begin{aligned} (-2) * 4 &= (-2)^2 - 4 \times 4^2 \\ &= 4 - 4 \times 16 \\ &= 4 - 64 \\ &= -60 \end{aligned}$$

Exercises

1. The formula below is used to convert temperatures in degrees Celsius to degrees Fahrenheit, where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.

$$F = 1.8\,C + 32$$

Find F if:

(a) $C = 10$ (b) $C = 20$ (c) $C = -10$

(d) $C = -5$ (e) $C = -20$ (f) $C = 15$

2. The formula

$$s = \frac{1}{2}(u + v)t$$

is used to calculate the distance, s, that an object travels if it starts with a velocity u and has a velocity v, t seconds later.

Find s if:

(a) $u = 2,\ v = 8,\ t = 2$ (b) $u = 3,\ v = 5,\ t = 10$

(c) $u = 1.2,\ v = 3.8,\ t = 4.5$ (d) $u = -4,\ v = 8,\ t = 2$

(e) $u = 4,\ v = -8,\ t = 5$ (f) $u = 1.6,\ v = 2.8,\ t = 3.2$

3. The length, l, of a spring is given by the formula

$$l = 20 - 0.08F$$

where F is the size of the force applied to the spring to compress it.

Find l if:

(a) $F = 5$ (b) $F = 20$

(c) $F = 24$ (d) $F = 15$

4. The formula

$$P = 120n - 400$$

gives the profit, P in \$, made when n phones are sold in a day at a shop.

Find P if:

(a) $n = 1$ (b) $n = 3$

(c) $n = 4$ (d) $n = 10$

How many phones must be sold to make a profit?

5. Work out the value of each function by substituting the values given, *without* using a calculator.

(a) $V = p^2 + q^2$
$p = 8$ and $q = 4$

(b) $p = a^2 - b^2$
$a = 10$ and $b = 7$

(c) $z = \sqrt{x + y}$
$x = 10$ and $y = 6$

(d) $Q = \sqrt{x - y}$
$x = 15$ and $y = 6$

(e) $P = \dfrac{x + y}{2}$
$x = 4$ and $y = -10$

(f) $Q = \sqrt{\dfrac{a}{b}}$
$a = 100$ and $b = 4$

(g) $V = \dfrac{x + 2y + z}{5}$
$x = 2$, $y = -5$ and $z = 8$

(h) $R = \dfrac{1}{a} + \dfrac{1}{b}$
$a = 4$ and $b = 2$

(i) $S = \dfrac{a}{b} + \dfrac{b}{c}$
$a = 3$, $b = 4$ and $c = 16$

(j) $R = 0.2a + 0.4b$
$a = 10$ and $b = 20$

(k) $T = \dfrac{a}{2} + \dfrac{b}{5}$
$a = -20$ and $b = 40$

(l) $C = \dfrac{ab}{a + b}$
$a = 10$ and $b = -5$

(m) $P = 2\sqrt{\dfrac{x^2}{y}}$
$x = 10$ and $y = 4$

(n) $A = \dfrac{ab^2}{c}$
$a = 2$, $b = 3$ and $c = 100$

(o) $X = \dfrac{b + c}{a}$
$a = 10$, $b = 1.7$ and $c = 2.1$

(p) $z = \sqrt{x^2 + y^2}$
$x = -3$ and $y = 4$

(q) $P = \sqrt{a^2 - b^2}$
$a = -10$ and $b = 6$

(r) $Q = \sqrt{x^2 + y^2 + z^2}$
$x = -10$, $y = 5$ and $z = 10$

6. Work out the value of each function by substituting the values given, using a calculator if necessary.

(a) $P = \dfrac{x - y}{z}$
$x = 10$, $y = 2.02$
and $z = 2.1$

(b) $V = \dfrac{x - y}{x + y}$
$x = 4.9$ and $y = 3.1$

(c) $R = \dfrac{x^2 - y^2}{4}$
$x = 3.6$ and $y = 1.6$

(d) $D = \dfrac{2}{x} + \dfrac{2}{y}$
$x = 0.4$ and $y = 0.8$

(e) $Q = \dfrac{x^2 + y^2}{5}$
$x = 3.7$ and $y = 5.9$

(f) $V = \dfrac{3x + 2y}{x + y}$
$x = 1.6$ and $y = 2.4$

7. If $a * b = a^2 - 4ab$, what is the value of $a * b$ when

   (a) $a = 2,\ b = -2$

   (b) $a = -3,\ b = -1$

8. The formula to convert temperatures from degrees Fahrenheit ($°F$) into degrees Celsius ($°C$) is

$$C = \frac{5}{9}\left(F - 32\right)$$

   Calculate the temperature in degrees Celsius which is equivalent to a temperature of $-4\ °F$.

9. Given that $a = 2$, $b = -3$ and $c = 0$, evaluate

   (i) $4a - 2b + 3c$

   (ii) a^c

10. Given that $m = \frac{1}{2}$, $p = \frac{3}{4}$, $t = -2$, calculate

   (a) $mp + t$ (b) $\frac{(m + p)}{t}$

Challenge!

There are 10 bank notes altogether. They consist of $10, $20 and $50 notes. If the total value of the notes is $180, find the number of each type of notes.

21.5 More Complex Formulae

Some formulae such as

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad \text{and} \quad z^2 = x^2 + y^2$$

arise in science or mathematics, but when used do not lead directly to values of f or z.

Here we show how to use the formula to calculate these values.

Worked Example 1

Use the formula

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

to find f if $u = 10$ and $v = 8$.

Solution

Substituting into the formula gives

$$\frac{1}{f} = \frac{1}{10} + \frac{1}{8}$$

First add together the two fractions using 40 as a common denominator:

$$\frac{1}{f} = \frac{4}{40} + \frac{5}{40}$$

$$\frac{1}{f} = \frac{9}{40}$$

Now to find f, turn both fractions upside-down to give

$$\frac{f}{1} = \frac{40}{9} \qquad \text{or} \qquad f = 4\frac{4}{9}$$

Worked Example 2

Find z using the formula

$$z^2 = x^2 + y^2$$

if $x = 3.6$ and $y = 4.8$.

Solution

Substituting these values into the formula gives

$$z^2 = 3.6^2 + 4.8^2$$

$$z^2 = 12.96 + 23.04$$

$$z^2 = 36$$

Now the square root can be taken of both sides to give

$$z = +\sqrt{36} \text{ or } -\sqrt{36}$$

$$z = 6 \text{ or } -6$$

Exercises

1. Use the formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

to find f if:

(a) $v = 3$ and $u = 4$ (b) $v = 6$ and $u = -5$

(c) $v = 7$ and $u = -3$ (d) $v = 10$ and $u = -4$

2. Find z using the formula

$$z^2 = x^2 + y^2$$

if:

(a) $x = 1.2$ and $y = 0.5$ (b) $x = 4.8$ and $y = 6.4$

(c) $x = 3$ and $y = 1.6$

3. Find the value of z as a fraction or mixed number in each case below.

(a) $\frac{1}{z} = \frac{x}{x+y}$
$x = 4$ and $y = -10$

(b) $\frac{1}{z} = \frac{x}{y} + \frac{y}{x}$
$x = 3$ and $y = 4$

(c) $\frac{1}{z} = \frac{2}{x} + \frac{3}{y}$
$x = 4$ and $y = -5$

(d) $\frac{1}{z} = \frac{x-y}{x+y}$
$x = -7$ and $y = -3$

(e) $\frac{1}{z} = \frac{x}{4} + \frac{3}{y}$
$x = 5$ and $y = -2$

(f) $\frac{1}{z} = \frac{1+x}{1-x}$
$x = 2$

(g) $\frac{1}{z} = \frac{x-2}{x+4}$
$x = \frac{1}{4}$

(h) $\frac{x+y}{x-y} = \frac{1}{z}$
$x = 4$ and $y = \frac{1}{2}$

(i) $\frac{x}{2} + \frac{3}{y} = \frac{1}{z}$
$x = 1$ and $y = 6$

4. Find z in each case below.

(a) $z^2 = 9 + x^2$
$x = 4$

(b) $z^2 = x + y$
$x = 147$ and $y = -3$

(c) $z^2 = x - y$
$x = 44$ and $y = -5$

(d) $z^2 = \frac{x}{y}$
$x = 363$ and $y = 3$

(e) $z^2 = \frac{x+6}{y}$
$x = 6$ and $y = 3$

(f) $z^2 = \frac{x}{8+y}$
$x = 16.9$ and $y = -7.9$

5. When three resistors are connected in parallel the total resistance R is given by

$$\frac{1}{R} = \frac{1}{X} + \frac{1}{Y} + \frac{1}{Z}$$

where X, Y and Z are the resistances of each resistor.

Find R if:

(a) $X = 10$, $Y = 20$ and $Z = 30$

(b) $X = 1000$, $Y = 5000$ and $Z = 2000$

(c) $X = 1500$, $Y = 2200$ and $Z = 1600$

6. Use the formula

$$y = \frac{x - 1}{\sqrt{\left(t - v^2\right)}}$$

to calculate the value of y given that

$x = 50$, $t = 2.5$ and $v = 0.6$

Give your answer correct to 1 decimal place.

Show all necessary working.

7. The formula $f = \dfrac{uv}{u + v}$ is used in the study of light.

(a) Calculate f when $u = 14.9$ and $v = -10.2$.
Give your answer correct to 3 significant figures.

(b) By rounding the values of u and v in part (a) to 2 significant figures, check whether your answer to part (a) is reasonable.
Show your working.

Investigation

Find four integers, a, b, c and d such that $a^3 + b^3 + c^3 = d^3$.

21.6 Changing the Subject

Sometimes a formula can be rearranged into a more useful format. For example, the formula

$$F = 1.8C + 32$$

can be used to convert temperatures in degrees Celsius to degrees Fahrenheit. It can be rearranged into the form

$$C = \ldots$$

to enable temperatures in degrees Fahrenheit to be converted to degrees Celsius. We say that the formula has been rearranged to make C the subject of the formula.

Worked Example 1

Rearrange the formula

$$F = 1.8C + 32$$

to make C the subject of the formula.

Solution

The aim is to remove all terms from the right hand side of the equation except for the C.

First subtract 32 from both sides, which gives

$$F - 32 = 1.8C$$

Then dividing both sides by 1.8 gives

$$\frac{F - 32}{1.8} = C$$

So the formula can be rearranged as

$$C = \frac{F - 32}{1.8}$$

Worked Example 2

The distance, s, travelled by a car in time t from initial speed u to final speed v is given by the formula

$$s = \frac{(u + v)t}{2}$$

Make v the subject of the formula.

Solution

First multiply both sides of the formula by 2 to give

$$2s = (u + v)t$$

Then divide both sides by t, to give

$$\frac{2s}{t} = u + v$$

Finally, subtract u from both sides to give

$$\frac{2s}{t} - u = v$$

So the formula becomes

$$v = \frac{2s}{t} - u$$

Exercises

1. Make x the subject of each of the following formulae.

(a) $y = 4x$ (b) $y = 2x + 3$ (c) $y = 4x - 8$

(d) $y = \dfrac{x+2}{4}$ (e) $y = \dfrac{x-2}{5}$ (f) $y = x + a$

(g) $y = \dfrac{x-b}{a}$ (h) $y = ax + c$ (i) $y = \dfrac{ax+b}{c}$

(j) $y = \dfrac{ax-c}{b}$ (k) $y = a + b + x$ (l) $y = \dfrac{x-a+b}{c}$

(m) $y = abx$ (n) $y = abx + c$ (o) $y = \dfrac{4ax-b}{3c}$

(p) $p = \dfrac{ax-bc}{d}$ (q) $y = (a+x)b$ (r) $y = \dfrac{(3+x)a}{4}$

(s) $q = \dfrac{3(x-4)}{2}$ (t) $v = \dfrac{5(x+y)}{4}$ (u) $z = a + \dfrac{(x-3)}{4}$

2. Ohm's law is used in electrical circuits and states that

$$V = IR$$

Write formulae with I and R as their subjects.

3. Newton's Second law states that $F = ma$.

Write formulae with m and a as their subjects.

4. The formula $C = 2\pi r$ can be used to find the circumference of a circle. Make r the subject of this formula.

5. The equation $v = u + at$ is used to find the velocities of objects.

(a) Make t the subject of this formula.

(b) Make a the subject of this formula.

6. The mean of three numbers x, y and z can be found using the formula

$$m = \frac{x+y+z}{3}$$

Make z the subject of this formula.

7. Make a the subject of the following formulae.

(a) $v^2 = u^2 + 2as$ (b) $s = at + \frac{1}{2}at^2$

8. The volume of a tin can is given by

$$V = \pi r^2 h$$

where r is the radius of the base and h is the height of the can.

(a) Make r the subject of the equation.

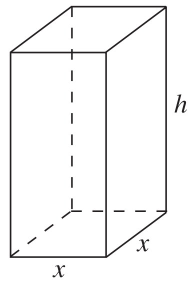
(b) Find r correct to 2 decimal places if $V = 250 \text{ cm}^3$ and $h = 10$ cm.

9. A box with a square base has its volume given by

$$V = x^2 h$$

and its surface area given by

$$A = 2x^2 + 4xh$$



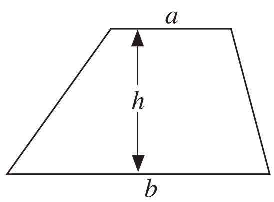
(a) Make h the subject of both formulae.

(b) Find h if $A = 24 \text{ cm}^2$ and $x = 2$ cm.

(c) Find h if $V = 250 \text{ cm}^3$ and $x = 10$ cm.

10. The area of a trapezium is given by

$$A = \frac{1}{2}(a + b)h$$



(a) Write the formula with a as its subject.

(b) In a particular trapezium $b = 2a$.
Use this to write a formula that does not involve b, and make a the subject.

11. (a) Average speed, v m s^{-1}, time, t seconds and distance, d metres, are related by the formula

$$d = vt.$$

Make v the subject of the formula.

(b) Use the formula to find the average speed for each of the following performances at the Beijing Olympics in 2008. Give your answers correct to 2 decimal places.

(i) Usain Bolt when he won the men's 100 m race in a time of 9.69 seconds

(ii) Usain Bolt when he won the men's 200 m race in a time of 19.30 seconds.

(iii) The Jamaican relay team for the men's 4×100 race in a time of 37.10 s.

Explain the differences in the average speeds.

21.7 More Change of Subject

This section uses some further approaches to rearranging formulae.

Worked Example 1

The period, T, of a pendulum of length, l, is given by the formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Make l the subject of the formula.

Solution

First divide both sides by 2π to give

$$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$$

Now the square root can be easily removed by squaring both sides of the equation, to give

$$\frac{T^2}{4\pi^2} = \frac{l}{g}$$

Finally, both sides can be multiplied by g to give

$$\frac{T^2 g}{4\pi^2} = l$$

so the rearranged formula is

$$l = \frac{T^2 g}{4\pi^2}$$

Worked Example 2

Make x the subject of the formula

$$y = 6 - 5x$$

Solution

To avoid leaving $-5x$ on the right hand side of the formula, first add $5x$ to both sides to give

$$y + 5x = 6$$

Then subtract y from both sides to give

$$5x = 6 - y$$

Finally, divide by 5 to give

$$x = \frac{6 - y}{5}$$

Worked Example 3

Make x the subject of the formula

$$q = \frac{1}{x} + \frac{1}{y}$$

Solution

First subtract $\frac{1}{y}$ from both sides so that the right hand side contains only terms involving x.

$$q - \frac{1}{y} = \frac{1}{x}$$

Now combine the two terms on the left hand side of the formula into a single fraction, by first making y the common denominator.

$$\frac{qy}{y} - \frac{1}{y} = \frac{1}{x}$$

$$\frac{qy - 1}{y} = \frac{1}{x}$$

Now both fractions can be turned upside-down to give

$$\frac{x}{1} = \frac{y}{qy - 1}$$

or

$$x = \frac{y}{qy - 1}$$

Exercises

1. Rearrange each of the following formulae so that x is the subject.

(a) $y = 5 - 3x$ (b) $y = 8 - 6x$ (c) $y = a - 2x$

(d) $y = \frac{6 - 2x}{5}$ (e) $y = \frac{8 - 7x}{2}$ (f) $y = \frac{7x - 5}{3}$

(g) $p = \frac{a - x - b}{2}$ (h) $q = \frac{8 - x + 2}{a}$ (i) $r = \frac{q - 5x}{b}$

2. For each formula below make a the subject.

(a) $q = \sqrt{\frac{a}{4}}$ (b) $z = \sqrt{\frac{a}{b}}$ (c) $z = \sqrt{\frac{c}{a}}$

(d) $y = 2\sqrt{\frac{2a}{3}}$ (e) $v = \frac{1}{4}\sqrt{\frac{a}{2b}}$ (f) $r = 5\sqrt{\frac{a}{\pi}}$

(g) $p = \sqrt{\frac{a + b}{4}}$ (h) $r = \frac{1}{2}\sqrt{\frac{b - a}{3}}$ (i) $c = 3\sqrt{\frac{2}{b + a}}$

3. Make u the subject of each of the following formulae.

(a) $a = \frac{1}{u} + \frac{1}{2}$ (b) $b = \frac{1}{u} - 2$ (c) $x = 2 - \frac{1}{u}$

(d) $\frac{1}{x} = \frac{1}{u} + \frac{1}{3}$ (e) $\frac{1}{p} = \frac{1}{u} - \frac{1}{5}$ (f) $\frac{1}{x} = \frac{2}{u} + \frac{1}{3}$

(g) $\frac{1}{r} = \frac{4}{u} + \frac{2}{v}$ (h) $\frac{1}{q} = \frac{1}{7} - \frac{1}{u}$ (i) $\frac{1}{p} = \frac{1}{a} - \frac{1}{u}$

4. The formula $T = 2\pi\sqrt{\frac{l}{g}}$ gives the time for a pendulum to complete one full swing.

(a) Make g the subject of the formula. (b) Find g if $l = 0.5$ and $T = 1.4$.

5. The formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ is used to find the focal length of a lens.

(a) Make v the subject of the formula. (b) Find v if $f = 12$ and $u = 8$.

6. If a ball is dropped from a height, h, it hits the ground with speed, v, given by

$$v = \sqrt{2gh}$$

(a) Make h the subject of this formula.

(b) Find h if $g = 10$ and $v = 6$.

(c) Make g the subject of the formula.

(d) Find the value of g on a planet when $h = 10$, $v = 4$.

7. A ball is thrown so that it initially travels at $45°$ to the horizontal. If it travels a distance R, then its initial speed, u, is given by

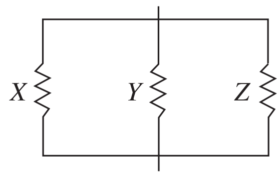
$$u = \sqrt{gR}$$

(a) Make R the subject of the formula.

(b) Find R if $u = 12$ and $g = 10$.

8. When three resistors with resistances X, Y and Z are connected as shown in the diagram, the total resistance is R, and

$$\frac{1}{R} = \frac{1}{X} + \frac{1}{Y} + \frac{1}{Z}$$



(a) Make X the subject of this equation.

(b) Find X if $R = 10$, $Y = 30$ and $Z = 40$.

9. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.

(a) Rearrange the formula to give r, in terms of V.

(b) Find the value of r when $V = 75$.