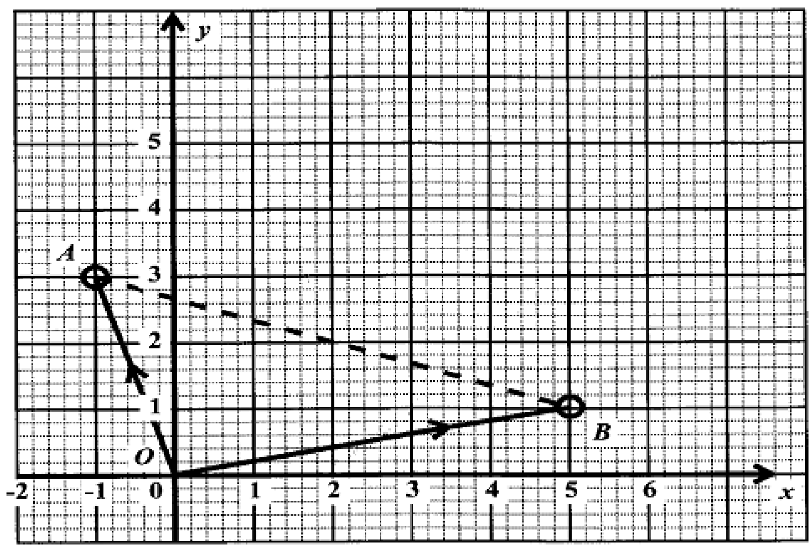
Vectors

Proving parallel and collinear/ class worksheet

(a) The diagram below shows two position vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$.



(i) Write as a column vector, in the form $\begin{pmatrix} x \\ y \end{pmatrix}$:

a) $\overrightarrow{OA}$ **(1 mark)**

b) $\overrightarrow{OB}$ **(1 mark)**

c) $\overrightarrow{BA}$ **(2 marks)**

(ii) Given that G is the mid-point of the line AB, write as a column vector in the form $\begin{pmatrix} x \\ y \end{pmatrix}$:

a) $\overrightarrow{BG}$ **(1 mark)**

b) $\overrightarrow{OG}$ **(1 mark)**

(a) The points A, B and C have position vectors $\overrightarrow{OA} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 12 \\ -2 \end{pmatrix}$ respectively.

(i) Express in the form $\begin{pmatrix} x \\ y \end{pmatrix}$ the vector

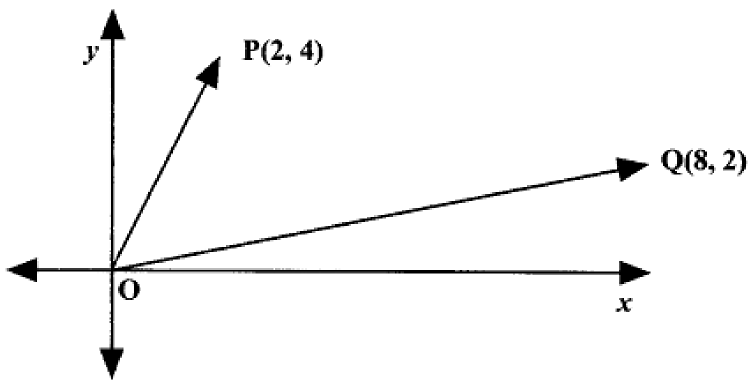
a) $\overrightarrow{BA}$ **(2 marks)**

b) $\overrightarrow{BC}$. **(2 marks)**

(ii) State ONE geometrical relationship between BA and BC. **(1 mark)**

(iii) Draw a sketch to show the relative positions of A, B and C. **(2 marks)**

In the diagram below, the coordinates of P and Q are (2, 4) and (8, 2) respectively. The line segment joining the origin (0, 0) to the point P may be written as $\overrightarrow{OP}$.



(i) What term is used to describe $\overrightarrow{OP}$? **(2 marks)**

(ii) Write EACH of the following in the form : $\begin{pmatrix} a \\ b \end{pmatrix}$

a) $\overrightarrow{OP}$ **(1 mark)**

b) $\overrightarrow{OQ}$ **(1 mark)**

c) $\overrightarrow{PQ}$ **(2 marks)**

(iii) Given that $\overrightarrow{OP} = \overrightarrow{RQ}$, determine the coordinates of the point, R. **(3 marks)**

(iv) State the type of quadrilateral formed by $PQRO$. **Justify your answer.** **(2 marks)**

(b) The position vectors of the points V, E and D relative to an origin O are

$$\overrightarrow{OV} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \ \overrightarrow{OE} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \ \overrightarrow{OD} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

respectively.

(i) Express the following vectors in the form $\begin{pmatrix} a \\ b \end{pmatrix}$:
$\overrightarrow{VE}$, $\overrightarrow{DV}$, $\overrightarrow{ED}$.

(ii) Prove that the points V, E and D lie on a straight line and show their relative positions on the line.

(iii) State the value of the ratio $EV{:}DV$. **(8 marks)**

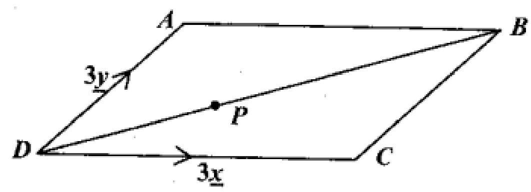
(b) The coordinates of the vertices of $\Delta\, PQS$ are $P(1, 5)$, Q (4, –1) and $S(6, 0)$.

(i) Write down the position vectors, $\overrightarrow{PQ}$ and $\overrightarrow{PS}$.

(ii) Determine the position vectors, $\overrightarrow{OG}$ and $\overrightarrow{OH}$, given that G and H are the midpoints of PQ and PS respectively.

(iii) Determine the vectors $\overrightarrow{GH}$ and $\overrightarrow{QS}$.

(iv) Hence, state TWO geometrical relationships between GH and QS. **(11 marks)**



In the figure above, **not drawn to scale**, $ABCD$ is a parallelogram such that $\overrightarrow{DC} = 3\underline{x}$ and $\overrightarrow{DA} = 3\underline{y}$. The point P is on DB such that $DP : PB = 1:2$.

(a) Express in terms of $\underline{x}$ and $\underline{y}$:

(i) $\overrightarrow{AB}$

(ii) $\overrightarrow{BD}$

(iii) $\overrightarrow{DP}$ **(5 marks)**

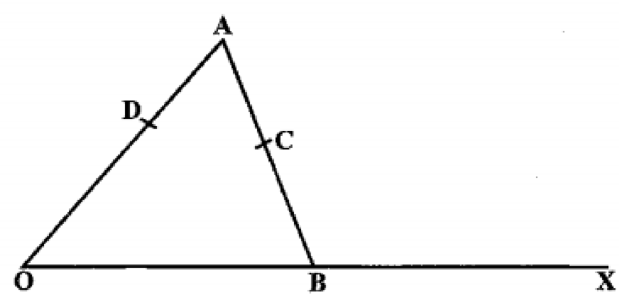
(b) Show that $\overrightarrow{AP} = \underline{x} - 2\underline{y}$. **(2 marks)**

(c) Given that E is the mid-point of DC, prove that A, P and E are collinear.
(4 marks)

(d) Given that $x = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ and $y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, use a vector method to prove that triangle AED is isosceles. **(4 marks)**

Total 15 marks

(b) In the diagram below, **not drawn to scale**, B is the midpoint of OX, C is the midpoint of AB, and D is such that $OD = 2DA$. The vectors **a** and **b** are such that $\overrightarrow{OA} = 3\mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.



(i) Write in terms of **a** and **b**:

a) $\overrightarrow{AB}$

b) $\overrightarrow{AC}$

c) $\overrightarrow{DC}$

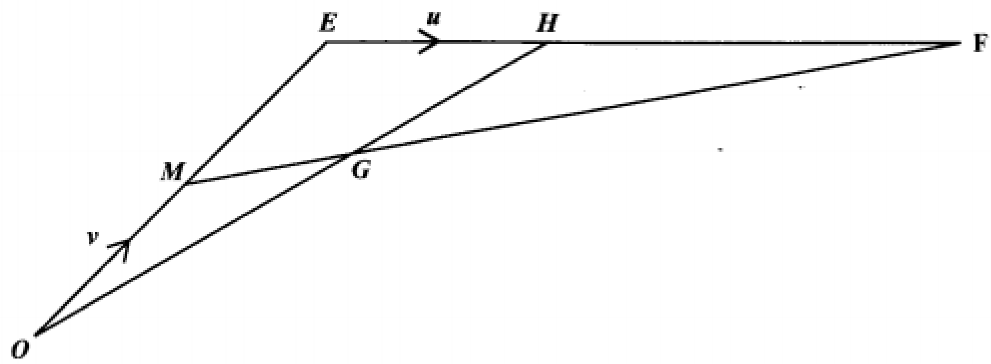
d) $\overrightarrow{DX}$ **(6 marks)**

(ii) State TWO geometrical relationships between DX and DC. **(2 marks)**

(iii) State ONE geometrical relationship between the points D, C, and X.
(1 mark)

(b) In the figure below, **not drawn to scale**, OE, EF and MF are straight lines. The point H is such that $EF = 3EH$. The point G is such that $MF = 5MG$. M is the midpoint of OE.

The vector $\overrightarrow{OM} = v$ and $\overrightarrow{EH} = u$.



(i) Write in terms of $\boldsymbol{u}$ and/or $\boldsymbol{v}$, an expression for:

a) $\overrightarrow{HF}$ **(1 mark)**

b) $\overrightarrow{MF}$ **(2 marks)**

c) $\overrightarrow{OH}$ **(2 marks)**

(ii) Show that $\overrightarrow{OG} = \frac{3}{5}\left(2v + u\right)$ **(2 marks)**

(iii) Hence, prove that O, G and H lie on a straight line. **(3 marks)**